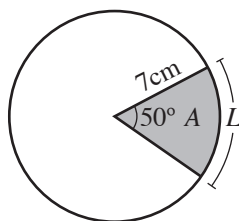


# MATH 1210 TEST 1. SPRING 2016

1. Evaluate the following expressions. Simplify compound fractions. You do not have to rationalize denominators. If the value is undefined, say so.

$$\begin{aligned} \tan(\pi/3) &= \\ \sin(5\pi/4) &= \\ \cos(\pi) &= \\ \sec(2\pi/3) &= \\ \tan(7\pi/6) &= \\ \csc(\pi/6) &= \\ \cos(3\pi/4) &= \\ \sin(3\pi/2) &= \\ \cot(5\pi/4) &= \\ \sec(5\pi/6) &= \\ \sin(0) &= \end{aligned}$$

2. Find the length  $L$  of the arc and the area  $A$  of the sector shown in the illustration.



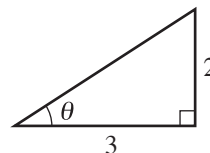
3. Convert the angle measures from degrees to radians (simplify your answers):

(a)  $75^\circ$     (b)  $210^\circ$

4. Convert the angle measures from radians to degrees:

(a)  $5\pi/3$     (b)  $\pi/2$

5. Give the values of the six trigonometric functions for the angle  $\theta$  shown in the illustration. Simplify compound fractions. You do not have to rationalize denominators.



6. If  $\sin \theta = 1/4$  and  $\cos \theta < 0$ , what are the values of the other five trigonometric functions for  $\theta$ ?

7. Evaluate the following expressions.

$$\begin{aligned} \sin^2(40^\circ) + \cos^2(40^\circ) &= \\ \sec(15^\circ) \cdot \cos(15^\circ) &= \\ \cos(-\pi/3) &= \\ 2 \sin(\pi/6) &= \\ \sin(2 \cdot \pi/6) &= \\ \cos\left(\frac{\pi}{6} + \frac{\pi}{2}\right) &= \\ \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{2}\right) &= \\ \csc(11\pi/4) &= \end{aligned}$$

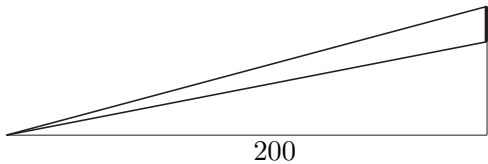
*For this portion of the test, you may use a calculator.*

8. (a) Convert  $14.7523^\circ$  to degree-minutes-seconds (DMS) notation.

(b) Convert  $123^\circ 37' 16''$  to a decimal degree. Use four decimal places of accuracy.

9. When the sun is  $20^\circ$  above the horizon, a tree casts an 80 foot long shadow. How tall is the tree?

10. When viewed from a distance of 200 feet, the angle of elevation to the bottom of a billboard is  $11^\circ$ , and to the top of the billboard is  $15^\circ$ . How tall is the billboard (not including the support poles it is built upon)?



## SOLUTIONS

1.

$$\tan(\pi/3) = \sqrt{3}$$

$$\sin(5\pi/4) = -\sqrt{2}/2$$

$$\cos(\pi) = -1$$

$$\sec(2\pi/3) = -2$$

$$\tan(7\pi/6) = 1/\sqrt{3}$$

$$\csc(\pi/6) = 2$$

$$\cos(3\pi/4) = -\sqrt{2}/2$$

$$\sin(3\pi/2) = -1$$

$$\cot(5\pi/4) = 1$$

$$\sec(5\pi/6) = -2/\sqrt{3}$$

$$\sin(0) = 0$$

2. Convert to radians

$$\theta = 50^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{18}$$

$$L = r\theta = 7 \cdot \frac{5\pi}{18} = \frac{35\pi}{18} \text{ cm.}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 7^2 \cdot \frac{5\pi}{18} = \frac{245\pi}{36} \text{ cm}^2.$$

3.

$$75^\circ = \frac{75\pi}{180} = \frac{5\pi}{12}$$

$$210^\circ = \frac{210\pi}{180} = \frac{7\pi}{6}$$

4.

$$\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$$

$$\frac{\pi}{2} \cdot \frac{180^\circ}{\pi} = 90^\circ$$

5. The length of the hypotenuse is  $\sqrt{2^2 + 3^2} =$

$\sqrt{13}$ . So

$$\sin \theta = 2/\sqrt{13}$$

$$\cos \theta = 2/\sqrt{13}$$

$$\tan \theta = 2/3$$

$$\csc \theta = \sqrt{13}/2$$

$$\sec \theta = \sqrt{13}/3$$

$$\cot \theta = 3/2$$

6. Use the Pythagorean identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1/16 + \cos^2 \theta = 1$$

$$\cos^2 \theta = 15/16$$

$$\cos \theta = -\sqrt{15}/4$$

Then

$$\csc \theta = 1/\sin \theta = 4$$

$$\sec \theta = 1/\cos \theta = -4\sqrt{15}$$

$$\tan \theta = \sin \theta / \cos \theta = -1/\sqrt{15}$$

$$\cot \theta = 1/\tan \theta = -\sqrt{15}$$

7.

$$\sin^2(40^\circ) + \cos^2(40^\circ) = 1$$

$$\sec(15^\circ) \cdot \cos(15^\circ) = (1/\cos 15^\circ) \cdot \cos 15^\circ = 1$$

$$\cos(-\pi/3) = \cos(\pi/3) = 1/2$$

$$2 \sin(\pi/6) = 2 \cdot (1/2) = 1$$

$$\sin(2 \cdot \pi/6) = \sin(\pi/3) = \sqrt{3}/2$$

$$\cos\left(\frac{\pi}{6} + \frac{\pi}{2}\right) = \cos(2\pi/3) = -1/2$$

$$\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{2}\right) = \sqrt{3}/2 + 0 = \sqrt{3}/2$$

$$\csc(11\pi/4) = \csc(3\pi/4) = \sqrt{2}$$

8. (a)  $14^\circ 45' 8''$  (b)  $123.6211^\circ$

9. If  $x$  is the height of the tree, then

$$\tan 20^\circ = \frac{x}{80} \implies x = 80 \tan(20^\circ) = 29.1 \text{ ft}$$

10.  $x$ : the distance from the ground to the top of the billboard

$$\tan 15^\circ = \frac{x}{200} \implies x = 200 \tan 15 = 53.6$$

$y$ : the distance from the ground to the bottom of the billboard

$$\tan 11^\circ = \frac{y}{200} \implies y = 200 \tan 11 = 38.9$$

The height of the billboard is  $y - x = 14.7 \text{ ft}$ .