Math 1210 Test 4. Fall 2013

1. Suppose that θ is in the second quadrant and that $\sin\theta = 1/4$. What is $\tan\theta$?

2. Find the exact values of each of the following expressions:

1. $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$
2. $\sin\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{4}\right)$
3. $\tan\left(\frac{\pi}{12}\right)$

3. Evaluate the following expressions (your final answer should not involve any trig. or inverse trig. functions).

1. $\tan(\tan^{-1}(1))$
2. $\sin^{-1}(\sin(2\pi/3))$
3. $\cos(\sin^{-1} x)$
4. $\cos(2\sin^{-1} x)$
5. $\cos(\sin^{-1}(2x))$

4. Find all real solutions to each of the following equations.

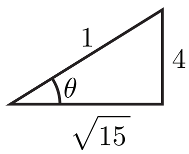
1. $2\cos\left(\theta+\frac{\pi}{3}\right)=1$
2. $4\sin^2\theta-3=0$
3. $4\sin^2\theta-4\sin\theta+1=0$
4. $\sin(2\theta)=\sin(4\theta)$

5. Verify each of the following identities. Be sure to clearly show all of your steps.

1. $\sec(2x)=\dfrac{\sec^2 x}{2-\sec^2 x}$
2. $\sin(4t)=4\sin t\cos t(1-2\sin^2 t)$
3. $\tan(\theta)=\csc(2\theta)-\cot(2\theta)$
4. $\cos\left(x-\dfrac{5\pi}{2}\right)=\sin x$

SOLUTIONS

1.



Since θ is in the second quadrant, tangent is negative, so

$$\tan\theta = -\frac{1}{\sqrt{15}}.$$

2.

1. $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$$= \sin(\pi/3)\cos(\pi/4) + \cos(\pi/3)\sin(\pi/4)$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

2. $\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{3}+\sqrt{2}}{2}$$

3. $\tan\left(\frac{\pi}{12}\right)$

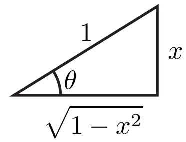
$$= \tan\left(\frac{\pi/6}{2}\right)$$
$$= \frac{1-\cos(\pi/6)}{\sin(\pi/6)}$$
$$= \frac{1-(\sqrt{3}/2)}{1/2}$$
$$= 2-\sqrt{3}$$

3.

1. $\tan(\tan^{-1}(1)) = 1$

2. $\sin^{-1}(\sin(2\pi/3)) = \sin^{-1}(\sqrt{3}/2) = \pi/3$

3. Put $\theta = \sin^{-1} x$ so that $\sin\theta = x$. Then



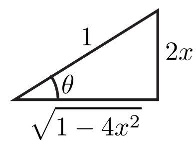
so

$$\cos(\sin^{-1} x) = \cos\theta = \sqrt{1-x^2}.$$

4. Put $\theta = \sin^{-1} x$ so that $\sin\theta = x$. Then (see the picture from the previous part)

$$\begin{aligned}\cos(2\sin^{-1} x) &= \cos(2\theta)\\ &= \cos^2\theta - \sin^2\theta\\ &= \left(\sqrt{1-x^2}\right)^2 - x^2\\ &= 1-2x^2.\end{aligned}$$

5. Put $\theta = \sin^{-1}(2x)$ so that $\sin\theta = 2x$. Then



so

$$\cos(\sin^{-1}(2x)) = \cos(\theta) = \sqrt{1-4x^2}.$$

4.

1.

$$\cos\left(\theta+\frac{\pi}{3}\right) = 1/2$$
$$\theta + \pi/3 = \begin{cases}\pi/3 + 2n\pi\\ 5\pi/3 + 2n\pi\end{cases}$$
$$\theta = \begin{cases}0 + 2n\pi\\ 4\pi/3 + 2n\pi\end{cases}$$

2.

$$\begin{aligned} \sin^2\theta &= 3/4 \\ \sin\theta &= \pm\sqrt{3}/2 \\ \theta &= \begin{cases} \pi/3 + 2n\pi \\ 2\pi/3 + 2n\pi \\ 4\pi/3 + 2n\pi \\ 5\pi/3 + 2n\pi \end{cases} \end{aligned}$$

3.

$$\begin{aligned} 4\sin^2\theta - 4\sin\theta + 1 &= 0 \\ (2\sin\theta - 1)(2\sin\theta - 1) &= 0 \\ \sin\theta &= 1/2 \\ \theta &= \begin{cases} \pi/6 + 2n\pi \\ 5\pi/6 + 2n\pi \end{cases} \end{aligned}$$

4.

$$\begin{aligned} \sin(2\theta) &= 2\sin(2\theta)\cos(2\theta) \\ 2\sin(2\theta)\cos(2\theta) - \sin(2\theta) &= 0 \\ \sin(2\theta)[2\cos(2\theta) - 1] &= 0 \end{aligned}$$

There are two factors. If $\sin(2\theta) = 0$, then

$$2\theta = \begin{cases} 0 + 2n\pi \\ \pi + 2n\pi \end{cases} \implies \theta = \begin{cases} 0 + n\pi \\ \pi/2 + n\pi \end{cases}.$$

If $\cos(2\theta) = 1/2$, then

$$2\theta = \begin{cases} \pi/3 + 2n\pi \\ 5\pi/3 + 2n\pi \end{cases} \implies \theta = \begin{cases} \pi/6 + n\pi \\ 5\pi/6 + n\pi \end{cases}.$$

5.

1.

$$\begin{aligned} \frac{\sec^2 x}{2 - \sec^2 x} &= \frac{\frac{1}{\cos^2 x}}{2 - \frac{1}{\cos^2 x}} \\ &= \frac{\frac{1}{\cos^2 x}}{\frac{2\cos^2 x - 1}{\cos^2 x}} \\ &= \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{2\cos^2 x - 1} \\ &= \frac{1}{2\cos^2 x - 1} \\ &= \frac{1}{\cos(2x)} \\ &= \sec(2x) \end{aligned}$$

2.

$$\begin{aligned} \sin(4t) &= 2\sin(2t)\cos(2t) \\ &= 2(2\sin t\cos t)(\cos^2 t - \sin^2 t) \\ &= 4\sin t\cos t(1 - \sin^2 t - \sin^2 t) \\ &= 4\sin t\cos t(1 - 2\sin^2 t) \end{aligned}$$

3.

$$\begin{aligned} \csc(2\theta) - \cot(2\theta) &= \frac{1}{\sin(2\theta)} - \frac{\cos(2\theta)}{\sin(2\theta)} \\ &= \frac{1 - \cos(2\theta)}{\sin(2\theta)} \\ &= \frac{1 - (\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta} \\ &= \frac{\sin^2\theta + \sin^2\theta}{2\sin\theta\cos\theta} \\ &= \frac{2\sin^2\theta}{2\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \end{aligned}$$

4.

$$
\begin{aligned}
&\cos\left(x - \frac{5\pi}{2}\right) \\
&\quad = \cos x \cos \frac{5\pi}{2} + \sin x \sin \frac{5\pi}{2} \\
&\quad = \cos x \cdot 0 + \sin x \cdot 1 \\
&\quad = \sin x
\end{aligned}
$$