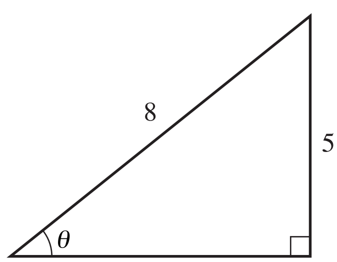
Math 1210 Test 1. Spring 2015

1. Evaluate the following trigonometric expressions. Give exact answers and simplify them.

$$\sin(\pi/3) =$$
$$\cos(3\pi/4) =$$
$$\tan(\pi) =$$
$$\sec(2\pi/3) =$$
$$\csc(5\pi/4) =$$
$$\cot(5\pi/3) =$$
$$\sin(11\pi/6) =$$
$$\sec(\pi) =$$
$$\tan(\pi/3) =$$
$$\sin(3\pi/2) =$$

2. Find the exact values of the six trigonometric functions of the angle θ shown in the triangle below. Simplify your answers and rationalize any denominators.



3. Suppose that $\cos\theta = 3/4$ and $\tan\theta = -\sqrt{7}/3$. Find the exact values of the other four trigonometric functions.

4. (a) If $\tan\theta = 1/3$, then

$$\tan\theta + \tan(\theta+\pi) + \tan(\theta+2\pi) =$$

(b) If $\sec\theta = 4$, then

$$\cos(\theta-2\pi) + \cos\theta + \cos(\theta+2\pi) =$$

(c) If $\cos\theta = 0.7$, then

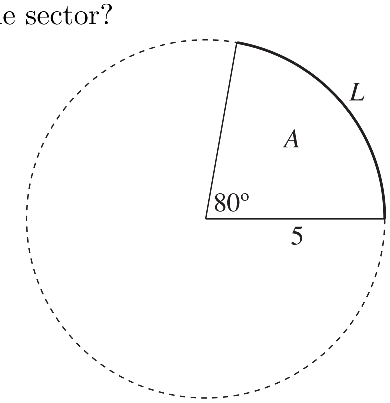
$$\cos\theta + \cos(-\theta) =$$

5. Use properties of trigonometric functions to evaluate the following expressions.

$$\sin(11\pi/3) =$$
$$\cos(-2\pi/3) =$$
$$\tan(\pi/5) + \tan(-\pi/5) =$$
$$\csc(21^\circ)\cdot\sin(21^\circ) =$$
$$\cot^2(7) - \csc^2(7) =$$
$$\cos(2\cdot 45^\circ) =$$
$$2\cdot\cos(45^\circ) =$$
$$\sin(\pi/3 + \pi/6) =$$
$$\sin(\pi/3) + \sin(\pi/6) =$$

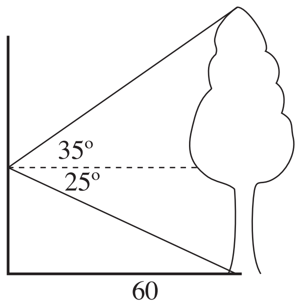
6. Convert 1.5 radians to degrees. Round your answer to the nearest degree.

7. (a) What is the length L of the arc shown in the illustration below? (b) What is the area A of the sector?



8. When the angle of elevation of sun is 55°, a tree casts a 35 foot long shadow. How tall is the tree?

9. When you look out the window of your third floor office, you can see a tree. The angle of depression to the bottom of the tree is 25°. The angle of elevation to the top of the tree is 35°. If the tree is sixty feet from the building, how tall is it?



SOLUTIONS

1.

$$\begin{aligned}\sin(\pi/3) &= \sqrt{3}/2\\ \cos(3\pi/4) &= -\sqrt{2}/2\\ \tan(\pi) &= 0\\ \sec(2\pi/3) &= -2\\ \csc(5\pi/4) &= -\sqrt{2}\\ \cot(5\pi/3) &= -\sqrt{3}/3\\ \sin(11\pi/6) &= -1/2\\ \sec(\pi) &= -1\\ \tan(\pi/3) &= \sqrt{3}\\ \sin(3\pi/2) &= -1\end{aligned}$$

2. Let a be the length of the third side of the triangle. Then

$$\begin{aligned}a^2+5^2 &= 8^2\\ a^2+25 &= 64\\ a^2 &= 39\\ a &= \sqrt{39}\end{aligned}$$

so

$$\begin{aligned}\sin\theta &= 5/8\\ \cos\theta &= \sqrt{39}/8\\ \tan\theta &= 5\sqrt{39}/39\\ \csc\theta &= 8/5\\ \sec\theta &= 8\sqrt{39}/39\\ \cot\theta &= \sqrt{39}/5\end{aligned}$$

3. First , we know that $\sin\theta/\cos\theta=\tan\theta$, so

$$\frac{\sin\theta}{3/4}=-\frac{\sqrt{7}}{3} \implies \sin\theta=-\frac{\sqrt{7}}{4}.$$

Then we can calculate

$$\csc\theta=\frac{1}{\sin\theta}=-\frac{4}{\sqrt{7}}=-\frac{4\sqrt{7}}{7},$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{4}{3}$$

and finally

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}.$$

4. (a) The tangent function is π periodic, so

$$\tan\theta+\tan(\theta+\pi)+\tan(\theta+2\pi) = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1.$$

(b) If $\sec\theta = 4$, then $\cos\theta = 1/4$. Since the cosine function is 2π periodic,

$$\cos(\theta-2\pi)+\cos\theta+\cos(\theta+2\pi) = \frac{1}{4}+\frac{1}{4}+\frac{1}{4} = \frac{3}{4}.$$

(c) Cosine is an even function, so if $\cos\theta = 0.7$, then

$$\cos\theta + \cos(-\theta) = 0.7 + 0.7 = 1.4.$$

5.

$$\sin(11\pi/3) = \sin(5\pi/3) = -\sqrt{3}/2$$

$$\cos(-2\pi/3) = \cos(2\pi/3) = -1/2$$

$$\tan(\pi/5) + \tan(-\pi/5) = \tan(\pi/5) - \tan(\pi/5) = 0$$

$$\csc(21^\circ)\cdot\sin(21^\circ) = \frac{1}{\sin(21^\circ)}\cdot\sin(21^\circ) = 1$$

$$\cot^2(7) - \csc^2(7) = \cot^2(7) - (1+\cot^2(7)) = 1$$

$$\cos(2\cdot 45^\circ) = \cos(90^\circ) = 0$$

$$2\cdot\cos(45^\circ) = 2(\sqrt{2}/2) = \sqrt{2}$$

$$\sin(\pi/3+\pi/6) = \sin(\pi/2) = 1$$

$$\sin(\pi/3)+\sin(\pi/6) = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

6.

$$1.5 \times \frac{180^\circ}{\pi} \approx 86^\circ.$$

7. Convert 80° to radians:

$$80^\circ \cdot \frac{\pi}{180^\circ} = \frac{4\pi}{9}$$

Then

$$L = r\theta = 5\cdot\frac{4\pi}{9} = \frac{20\pi}{9} \approx 6.98$$

and

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}5^2\cdot\frac{4\pi}{9} = \frac{50\pi}{9} \approx 17.45$$

8. Let x be the height of the tree. Then

$$\tan 55^\circ = x/35$$
$$x = 35\tan 55^\circ$$
$$x \approx 50 \text{ feet}$$

9. The horizontal line divides the picture into two right triangles. Let x be the length of the side opposite the 35° angle, and y be the length of the side opposite the 25° angle. Then

$$\tan 35^\circ = \frac{x}{60} \implies x = 60\tan 35^\circ \approx 42$$

and

$$\tan 25^\circ = \frac{y}{60} \implies y = 60\tan 25^\circ \approx 28.$$

The height of the tree is then $42 + 28 = 70$ feet.