Math 1210 Test 3. Spring 2014

1. Evaluate the following expressions:

(a) $\sin^{-1}(1) =$

(b) $\cos^{-1}(0) =$

(c) $\tan^{-1}(\sqrt{3}) =$

(d) $\sec^{-1}(2) =$

2. Evaluate the following expressions:

(a) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) =$

(b) $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right) =$

(c) $\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right) =$

(d) $\cos\left(\sin^{-1}(x)\right) =$

3. Use the addition, subtraction, double angle, and half angle formulas to answer the following questions.

(a) If $\sin\theta = 3/5$ and $\pi/2 < \theta < \pi$, what is $\sin(2\theta)$?

(b) Find the exact value of $\sin(105^\circ)$.

(c) If $\cos\theta = 4/5$ and θ is in Quadrant I, what is $\cos(\theta + \pi/2)$?

4. Solve the following equations (give all real solutions).

(a) $2\sin(3\theta) + 1 = 0$

(b) $5\csc\theta - 3 = 2$

(c) $\cos^2\theta - \sin^2\theta + \sin\theta = 0$

(d) $\cos(2\theta) + 6\sin^2\theta = 4$

5. Verify the following identities. Clearly show your steps.

(a) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 2\sec^2\theta$

(b) $\dfrac{\cos(a-b)}{\sin a\cos b} = \cot a + \tan b$

(c) $\dfrac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$

1.

(a) $\sin^{-1}(1) = \pi/2$

(b) $\cos^{-1}(0) = \pi/2$

(c) $\tan^{-1}(\sqrt{3}) = \pi/3$

(d) $\sec^{-1}(2) = \pi/3$

2.

(a) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = 1/2$

(b) $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right) = 3\pi/4$

(c) $\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \sqrt{5}/3$

(d) $\cos\left(\sin^{-1}(x)\right) = \sqrt{1-x^2}$

3. (a)

$$\begin{aligned}\sin(2\theta) &= 2\sin\theta\cos\theta \\ &= 2(3/5)(-4/5) = -24/26\end{aligned}$$

(b)

$$\begin{aligned}\sin(105^\circ) &= \sin(60^\circ + 45^\circ) \\ &= \sin(60^\circ)\cos(45^\circ) + \sin(45^\circ)\cos(60^\circ) \\ &= (\sqrt{3}/2)(\sqrt{2}/2) + (\sqrt{2}/2)(1/2) \\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

(c)

$$\begin{aligned}\cos(\theta + \pi/2) &= \cos\theta\cos(\pi/2) - \sin\theta\sin(\pi/2) \\ &= \cos\theta \cdot 0 - \sin\theta \cdot 1 \\ &= -\sin\theta \\ &= -3/5\end{aligned}$$

4. (a)

$$\begin{gathered}2\sin(3\theta) + 1 = 0 \\ \sin(3\theta) = -1/2 \\ 3\theta = \begin{cases}7\pi/6 + 2n\pi \\ 11\pi/6 + 2n\pi\end{cases} \\ \theta = \begin{cases}7\pi/18 + 2n\pi/3 \\ 11\pi/18 + 2n\pi/3\end{cases}\end{gathered}$$

(b)

$$\begin{gathered}5\csc\theta - 3 = 2 \\ 5\csc\theta = 5 \\ \csc\theta = 1 \\ \sin\theta = 1 \\ \theta = \pi/2 + 2n\pi\end{gathered}$$

(c)

$$\begin{gathered}\cos^2\theta - \sin^2\theta + \sin\theta = 0 \\ 1 - \sin^2\theta - \sin^2\theta + \sin\theta = 0 \\ -2\sin^2\theta + \sin\theta + 1 = 0 \\ 2\sin^2\theta - \sin\theta - 1 = 0 \\ (2\sin\theta + 1)(\sin\theta - 1) = 0 \\ \sin\theta = -1/2 \quad \text{or} \quad \sin\theta = 1\end{gathered}$$

When $\sin\theta = -1/2$,

$$\theta = \begin{cases}7\pi/6 + 2n\pi \\ 11\pi/6 + 2n\pi\end{cases}$$

When $\sin\theta = 1$,

$$\theta = \pi/2 + 2n\pi.$$

(d)

$$\cos(2\theta) + 6\sin^2\theta = 4$$
$$1 - 2\sin^2\theta + 6\sin^2\theta = 4$$
$$4\sin^2\theta = 3$$
$$\sin^2\theta = 3/4$$
$$\sin\theta = \pm\sqrt{3}/2$$
$$\theta = \begin{cases} \pi/3 + 2n\pi \\ 2\pi/3 + 2n\pi \\ 4\pi/3 + 2n\pi \\ 5\pi/3 + 2n\pi \end{cases}$$

5. (a)

$$\begin{aligned} &\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} \\ &= \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} \\ &= \frac{2}{1-\sin^2\theta} \\ &= \frac{2}{\cos^2\theta} \\ &= 2\sec^2\theta \end{aligned}$$

(b)

$$\begin{aligned} &\frac{\cos(a-b)}{\sin a\cos b} \\ &= \frac{\cos a\cos b + \sin a\sin b}{\sin a\cos b} \\ &= \frac{\cos a\cos b}{\sin a\cos b} + \frac{\sin a\sin b}{\sin a\cos b} \\ &= \frac{\cos a}{\sin a} + \frac{\sin b}{\cos b} \\ &= \cot a + \tan b \end{aligned}$$

(c)

$$\begin{aligned} &\frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} \\ &= \frac{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}} \\ &= \frac{(\cos^2\theta - \sin^2\theta)/(\sin\theta\cos\theta)}{\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}} \\ &= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \cdot \frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta} \\ &= \cos^2\theta - \sin^2\theta \\ &= \cos(2\theta) \end{aligned}$$