Math 1210. Practice for Test 2

1. Give the exact values of the following expressions. Simplify your answer.

(a) $\sin(4\pi/3)$

(b) $\cos(5\pi/6)$

(c) $\tan(\pi/3)$

(d) $\csc(11\pi/6)$

(e) $\cot(5\pi/4)$

(f) $\sec(\pi/2)$

(g) $\tan(\pi)$

2. Use properties of the trigonometric functions to answer the following questions.

(a) If $\sin\theta = 0.4$, what is $\sin(-\theta) + \sin(\theta + 2\pi)$?

(b) If $\sin\theta = 1/3$ and $\cos\theta > 0$, what is $\tan\theta$?

(c) Evaluate $\tan(\pi/6 + \pi/6)$.

(d) Evaluate $\tan(\pi/6) + \tan(\pi/6)$

(e) If $\cos\theta = 1/5$, what is $\cos\theta + \cos(-\theta)$?

(f) If $\sin\theta = 1/5$, what is $\sin\theta + \sin(-\theta)$?

(g) What is the smallest positive value that $\sec\theta$ can be?

3. Sketch the graphs of the following functions

(a) $f(x) = 2\sin x + 1$

(b) $f(x) = -\cos(2x)$

4. Find the period, amplitude and phase shift. Use this information to sketch the graph of the equation.

(a) $y = 2\cos(x + \pi/3)$

(b) $y = \sin(3x + \pi)$

(c) $y = 3\sin\left(\frac{\pi x}{2} + \pi\right)$

5. Sketch the graph of the functions

(a) $y = \tan(x + \pi/4)$

(b) $y = -2\sec(x)$

6. Evaluate

(a) $\cot^{-1}(\sqrt{3})$

(b) $\csc^{-1}(-1)$

(c) $\cos^{-1}(\cos(4\pi/5))$

(d) $\sin(\sin^{-1}(1.5))$

(e) $\tan^{-1}(\tan(-2\pi/3))$

(f) $\sec(\cos^{-1}(1/2))$

(g) $\tan(\cos^{-1}(1/3))$

(h) $\csc(\tan^{-1}(2))$

7. Write an equivalent algebraic expression (with no trigonometry) for each of the following:

(a) $\cos(\tan^{-1}(u))$

(b) $\sin(\cos^{-1}(2u))$

1.

(a) $\sin(4\pi/3) = -\frac{\sqrt{3}}{2}$

(b) $\cos(5\pi/6) = -\frac{\sqrt{3}}{2}$

(c) $\tan(\pi/3) = \sqrt{3}$

(d) $\csc(11\pi/6) = -2$

(e) $\cot(5\pi/4) = 1$

(f) $\sec(\pi/2) =$ undefined

(g) $\tan(\pi) = 0$

2. (a) Because sine is odd, $\sin(-\theta) = -\sin(\theta)$ and because it is 2π-periodic, $\sin(\theta + 2\pi) = \sin(\theta)$. Therefore

$$\sin(-\theta) + \sin(\theta + 2\pi) = -\sin(\theta) + \sin(\theta) = 0.$$

(b) Use the Pythagorean identity.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ (1/3)^2 + \cos^2\theta &= 1 \\ 1/9 + \cos^2\theta &= 1 \\ \cos^2\theta &= 8/9 \\ \cos\theta &= \pm\sqrt{8/9} \end{aligned}$$

Note that we are told that $\cos\theta$ is positive, so $\cos\theta = \sqrt{8}/3$. Then

$$\tan\theta = \frac{1/3}{\sqrt{8}/3} = \frac{1}{\sqrt{8}} = \frac{\sqrt{8}}{8}.$$

(c)

$$\tan(\pi/6 + \pi/6) = \tan(\pi/3) = \sqrt{3}.$$

(d)

$$\tan(\pi/6) + \tan(\pi/6) = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

(e) Cosine is an even function, so

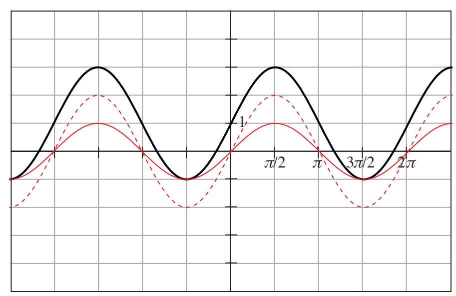
$$\cos\theta + \cos(-\theta) = \cos\theta + \cos\theta = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}.$$

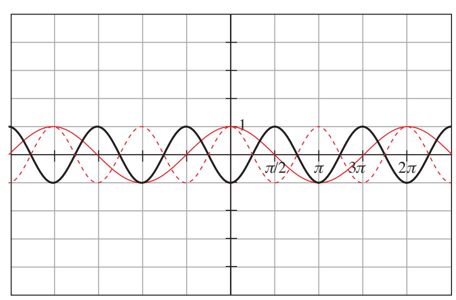
(f) Sine is an odd function so

$$\sin\theta + \sin(-\theta) = \sin\theta - \sin\theta = 0.$$

(g) Remember that $\sec\theta = 1/\cos\theta$. The maximum positive value that $\cos\theta$ can be is one, so the minimum positive value that $\sec\theta$ can be is $1/1 = 1$. You can also see this by looking at the graph of $\sec\theta$.

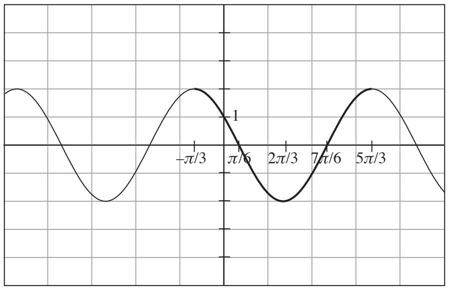
3. The light red curve is the initial trig. function. The dotted curve is an intermediate step in the transformation, and the black curve is the final graph.



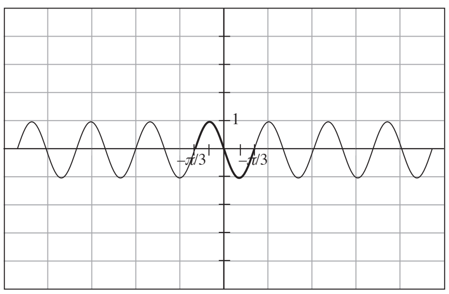


4. One period is shown in with the thick black line.

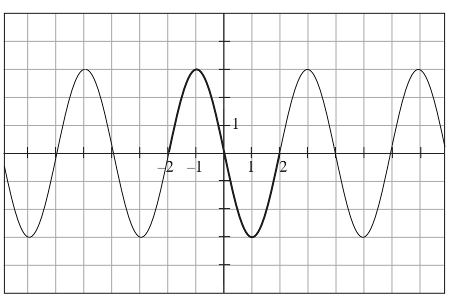
(a) The amplitude is 2, the period is 2π and the phase shift is $-\pi/3$.



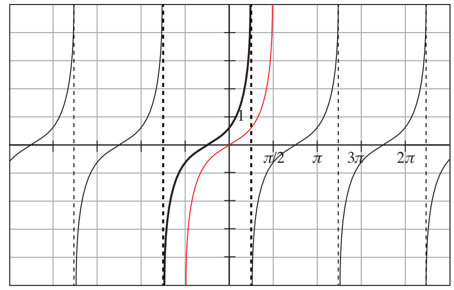
(b) The amplitude is 1, the period is $2\pi/3$, and the phase shift is $-\pi/3$.

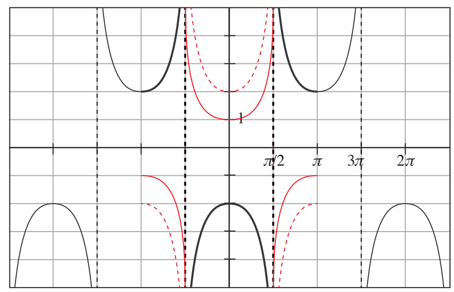


(c) The amplitude is 3, the period is $2\pi/(\pi/2) = 4$, and the phase shift is $-\pi/(\pi/2) = -2$.



5. The red curve is one period of the original trig function. The black is its transformation (the thick black line shows one period of it).





6.

(a) $\cot^{-1}(\sqrt{3}) = \pi/6$

(b) $\csc^{-1}(-1) = -\pi/2$

(c) $\cos^{-1}(\cos(4\pi/5)) = 4\pi/5$

(d) $\sin(\sin^{-1}(1.5)) = DNE$

(e) $\tan^{-1}(\tan(-2\pi/3)) = \pi/3$

(f) $\sec(\cos^{-1}(1/2)) = 2$

(g) $\tan(\cos^{-1}(1/3)) = \sqrt{8}$

(h) $\csc(\tan^{-1}(2)) = \sqrt{5}/2$

7.

(a) $\cos(\tan^{-1}(u)) = \dfrac{1}{\sqrt{1+u^2}}$

(b) $\sin(\cos^{-1}(2u)) = \sqrt{1-4u^2}$