Math 2040 Test 2. Fall 2016

1. Use the limit definition of the derivative to compute the derivative of the function $f(x) = 3x^2 + 2x$ at the point $a = 2$. No credit will be given for any other method.

2. Use the limit definition of the derivative to compute the derivative of the function $f(x) = \sqrt{3x+5}$. No credit will be given for any other method.

3. Find the equation of the tangent line to the curve $y = \cos x + 5x^2$ at the point $(0, 1)$.

4. Evaluate the limit: $\lim_{x\to 0} \dfrac{\sin(4x)}{3x}$.

For problems 5–12, compute the derivative of $f(x)$. Simplify your answer.

5. $f(x) = x^3 + \sqrt{x} + e^3$

6. $f(x) = \dfrac{x+1}{x^2+1}$

7. $f(x) = \dfrac{e^x}{1+e^x}$

8. $f(x) = x \sin x \cos x$

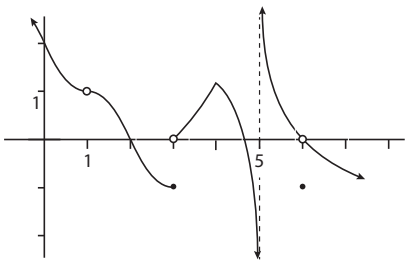
9. $f(x) = \dfrac{1}{\sqrt{x^2+2x}}$

10. $f(x) = (\sec x + 1)^3$

11. $f(x) = \sec^2(x^3)$

12. $f(x) = \left(\dfrac{x^3+1}{x^2+4}\right)^5$

13. The graph of $y = f(x)$ is shown below.

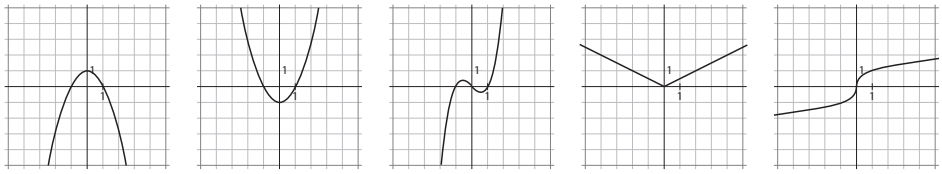


(a) List all values a for which $f(a)$ fails to exist.

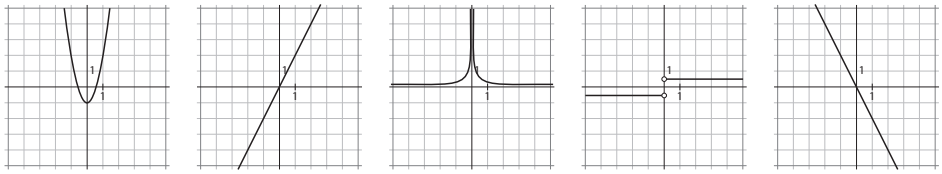
(b) List all values a for which $\lim_{x\to a} f(x)$ fails to exist.

(c) List all values a for which $f'(a)$ fails to exist.

14. Match the function on the top row with its derivative on the bottom row.



1 2 3 4 5



A B C D E

$1 \mapsto$ $2 \mapsto$ $3 \mapsto$ $4 \mapsto$ $5 \mapsto$

SOLUTIONS

1.

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \frac{3(2+h)^2+2(2+h)-(12+4)}{h} \\ &= \lim_{h\to 0} \frac{12+12h+3h^2+4+2h-16}{h} \\ &= \lim_{h\to 0} \frac{h(14+3h)}{h} \\ &= \lim_{h\to 0} (14+3h) \\ &= 14. \end{aligned}$$

2.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{3(x+h)+5}-\sqrt{3x+5}}{h} \end{aligned}$$

Multiply the numerator and denominator by the conjugate

$$\sqrt{3(x+h)+5}+\sqrt{3x+5}$$

to get

$$\begin{aligned} &= \lim_{h\to 0} \frac{3x+3h+5-3x-5}{h(\sqrt{3(x+h)+5}+\sqrt{3x+5})} \\ &= \lim_{h\to 0} \frac{3h}{h(\sqrt{3(x+h)+5}+\sqrt{3x+5})} \\ &= \lim_{h\to 0} \frac{3}{\sqrt{3(x+h)+5}+\sqrt{3x+5}} \\ &= \frac{3}{2\sqrt{3x+5}} \end{aligned}$$

3. $dy/dx = \sin x + 5x$, so the slope of the tangent line at $x = 0$ is $m = -\sin(0) + 0 = 0$. The tangent line is $y = 1$.

4.

$$\lim_{x\to 0} \frac{\sin(4x)}{3x} = \lim_{4x\to 0} \frac{\sin(4x)}{4x} \cdot \frac{4}{3} = 1 \cdot \frac{4}{3} = \frac{4}{3}.$$

5.

$$f'(x) = 3x^2 + \frac{1}{2}x^{-1/2} = 3x^2 + \frac{1}{2\sqrt{x}}.$$

6.

$$\begin{aligned} f'(x) &= \frac{(x^2+1)(1)-(x+1)(2x)}{(x^2+1)^2} \\ &= \frac{x^2+1-2x^2-2x}{(x^2+1)^2} \\ &= \frac{-x^2-2x+1}{(x^2+1)^2} \end{aligned}$$

7.

$$f'(x) = \frac{(1+e^x)e^x - e^x(e^x)}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

8.

$$\begin{aligned} f'(x) &= x(-\sin^2 x + \cos^2 x) + \sin x \cos x \\ &= x\cos^2 x - x\sin^2 x + \sin x \cos x \end{aligned}$$

9.

$$\begin{aligned} f'(x) &= -\frac{1}{2}(x^2+2x)^{-3/2} \cdot (2x+2) \\ &= \frac{-(x+1)}{(x^2+2x)^{3/2}} \end{aligned}$$

10.

$$f'(x) = 3(\sec x + 1)^2 \cdot \sec x \tan x.$$

11.

$$\begin{aligned} f'(x) &= 2\sec(x^3) \cdot \sec(x^3)\tan(x^3) \cdot 3x^2 \\ &= 6x^2 \sec^2(x^3)\tan(x^3) \end{aligned}$$

12.

$$\begin{aligned} f'(x) &= 5\left(\frac{x^3+1}{x^2+4}\right)^4 \cdot \frac{(x^2+4)(3x^2)-(x^3+1)(2x)}{(x^2+4)^2} \\ &= \frac{5(x^3+1)^4(x^4+12x^2-2x)}{(x^2+4)^6} \end{aligned}$$

13. (a) a=1,5 (b) 3,5 (c) 1,3,4,5,6

14. $1 \mapsto E$, $2 \mapsto B$, $3 \mapsto A$, $4 \mapsto D$, $5 \mapsto C$