MATH 2050 TEST 2. SPRING 2016

1. (a) An object has a mass of $5\,kg$. How much work is required to lift it $10\,m$? (b) An object has a weight of $5\,lb$. How much work is required to lift it $10\,ft$? (c) What is the volume of a cylindrical shell with a radius r, height h, and thickness Δx?

2. Find the area of the region between the curve $x = y^2$ and the line $y = 2 - x$.

3. Set up an integral (or integrals) to calculate the area of the region bounded by $y = x^2$, $y = 4$, and $y = 9$. You do not have to evaluate the integral(s).

4. The region between $y = x$ and $y = \sqrt{x}$ is revolved around the line $x = 2$. Use the disk/washer method to set up an integral (or integrals) to calculate the volume of the resulting solid. You do not have to evaluate the integral(s).

5. The region between $y = 4 - x^2$ and $y = 0$ is revolved around the line $x = -4$. Use the shell method to set up an integral (or integrals) to calculate the volume of the resulting solid. You do not have to evaluate the integral(s).

6. The base of a solid is a triangle with vertices $(0, 0)$, $(2, 0)$ and $(0, 4)$. Cross sections taken parallel to the x-axis are isosceles triangles whose altitudes are the same length as their bases. Set up an integral to calculate the volume of the solid. You do not have to evaluate the integral.

7. A 30 foot long rope weighing 10 pounds hangs down in a well. A 40 pound bucket of water is attached to the end of the rope. Set up an integral (or integrals) to calculate the work required to lift the bucket and rope to the top of the well. You do not have to evaluate the integral(s).

8. A water tank is in the shape of an inverted cone (the point of the cone is at the bottom of the tank). It has a radius of 3 meters and a height of 5 meters. Assuming that the tank is completely full, set up an integral to calculate the work required to completely empty the tank through a pipe at its top. You do not have to evaluate the integral. Recall that the density of water is $1000kg/m^3$ and that the force of gravity is $9.81m/s^2$.

SOLUTIONS

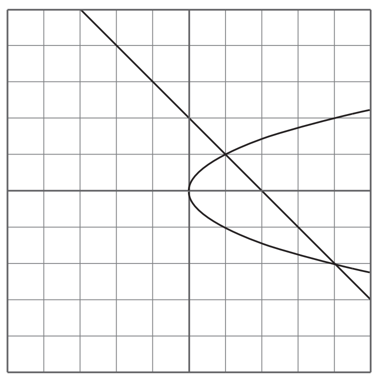
1.

(a) $W = 9.81 \cdot 5 \cdot 10 J$

(b) $W = 5 \cdot 10 ft \cdot lb$

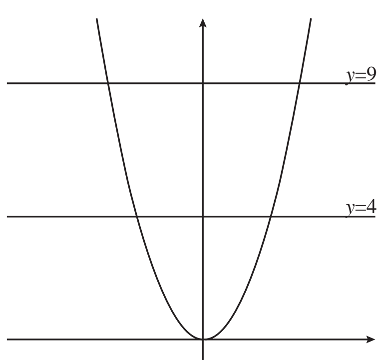
(c) $V = 2\pi r h \Delta x$

2.



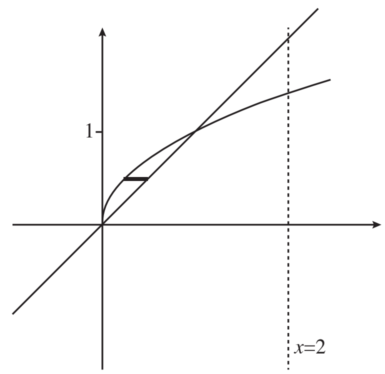
$$\begin{aligned} A &= \int_{-1}^{2} (2-y) - y^2\, dy \\ &= \left(2y - \frac{y^2}{2} - \frac{y^3}{3}\right)\Bigg|_{-2}^{1} \\ &= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) \\ &= 9/2 \end{aligned}$$

3.



$$A = \int_4^9 2\sqrt{y}\, dy$$

4.



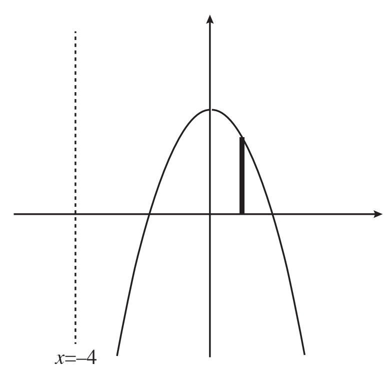
The volume of one washer is

$$V(\text{washer}) = \pi\left((2-y^2)^2 - (2-y)^2\right)\, \Delta y.$$

The total volume of the solid is

$$V = \int_0^1 \pi\left((2-y^2)^2 - (2-y)^2\right)\, dy.$$

5.



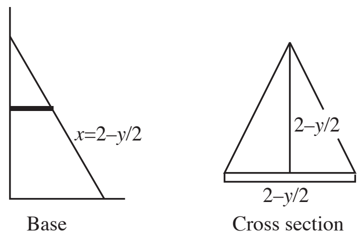
The volume of one shell is

$$V(\text{shell}) = 2\pi(x+4)(4-x^2)\, \Delta x.$$

The total volume is

$$V = \int_{-2}^{2} 2\pi(x+4)(4-x^2)\, dx.$$

6.



The width of a cross section is $x = 2 - y/2$ (and the height is the same), so the volume of one cross section is

$$V(\times\text{section}) = \frac{1}{2}\,(2 - y/2)^2\ \Delta y.$$

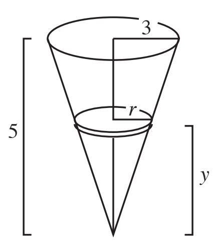
The total volume is

$$V = \int_0^4 \frac{1}{2}(2 - y/2)^2\, dy.$$

7. When the bucket has been pulled up a distance x from the bottom, it and the rope weigh $40 + \frac{1}{3} \cdot (30 - x)$. The total work to lift it is then

$$W = \int_0^{30} 40 + \frac{1}{3} \cdot (30 - x)\, dx = \int_0^{30} (50 - x/3)\, dx.$$

8.



By similar triangles, the radius r of the disk at height y is given by $r = (3/5)y$. The volume of that disk is

$$V = \pi(3y/5)^2\ Deltay$$

so the force of gravity on it is

$$F = 9810\pi(3y/5)^2\ Deltay$$

and the vertical distance it has to travel to get to the pipe is $5 - y$. The work to completely empty the cone is then

$$W = \int_0^5 9810\pi(3y/5)^2(5 - y)\, dy.$$