Math 2050 Test 4. Spring 2016

1. Find the length of the curve $y = \ln(\sec x)$, $0 \le x \le \pi/3$.

2. Set up (and simplify if possible) an integral to compute the area of the surface formed by revolving $y = \cos x$, $0 \le x \le \pi$ around the x axis. You do not have to evaluate the integral.

3. (a) Write parametric equations for the line that passes through the point $(2, 2)$ and $(3, 5)$.

(b) Write parametric equation for the circle that is centered at $(3, 2)$ and has radius 4.

4. A particle is moving along a path given by the parametric equations:

$$x(t) = \ln(t^3), \quad y(t) = \ln(t), \quad (t > 0).$$

(a) De-parametrize the path (write and equation that relates x directly to y).

(b) Sketch its graph, and draw an arrow on it to indicate which direction the particle is moving.

5. The curve $x = \cos t$, $y = \sin t \cos t$, $(0 \le t < 2\pi)$ crosses through $(0, 0)$ twice. Find the slopes of the tangent lines at those two crossings.

6. (a) The rectangular coordinates for a point are $(1, \sqrt{3})$. Find polar coordinates for it.

(b) Polar coordinates for a point are $(-2, \pi/2)$. Find rectangular coordinates for it.

7. Write the polar equation $r = 4 \sin\theta$ as a Cartesian equation. Graph it.

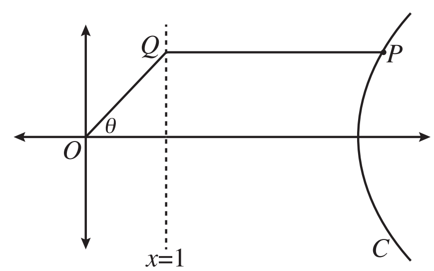
8. Sketch the graph of the polar curve $r = 1 - \sin(\theta)$.

9. Sketch the graph of the polar curve $r = 3\cos(2\theta)$.

10. Set up an integral to calculate the area inside one loop of the petal curve $r = 2\cos(3\theta)$. You do not have to evaluate the integral.

11. Set up an integral (or integrals) to find the area that lies inside the polar curve $r = 3\cos\theta$ but outside $r = 2 - \cos\theta$. You do not have to evaluate the integral.

12. A curve $\mathcal{C}$ is constructed as follows: starting from the origin O, rotate an angle θ and move in a straight line until intersecting the line $x = 1$ at the point Q. Now travel horizontally to P so that the distance from P to Q is the square of the distance from O to Q. Parametrize, with parameter θ, the curve $\mathcal{C}$.



SOLUTIONS

1. Since $dy/dx = \tan x$,

$$\begin{aligned} L &= \int_0^{\pi/3} \sqrt{1+\tan^2 x}\, dx \\ &= \int_0^{\pi/3} \sec x\, dx \\ &= \ln|\sec x + \tan x|\Big|_0^{\pi/3} \\ &= \ln|2+\sqrt{3}| \end{aligned}$$

2. Since $dy/dx = -\sin x$,

$$A = \int_0^{\pi} 2\pi \cos x \sqrt{1+\sin^2 x}\, dx$$

3. (a)

$$\begin{cases} x(t) = 2 + t \\ y(t) = 2 + 3t \end{cases}$$

(b)

$$\begin{cases} x(t) = 3 + 4\cos t \\ y(t) = 2 + 4\sin t \end{cases}$$

4. (a) Write $x = 3\ln t3 = y$, so $y = x/3$.

(b)

5. When $x = y = 0$, $t = \pi/2$ or $t = 3\pi/2$. Then

$$m = \frac{y'(t)}{x'(t)} = \frac{-\sin^2 t + \cos^2 t}{-\sin t}$$

At $t = \pi/2$, $m = 1$. At $t = 3\pi/2$, $m = -1$.

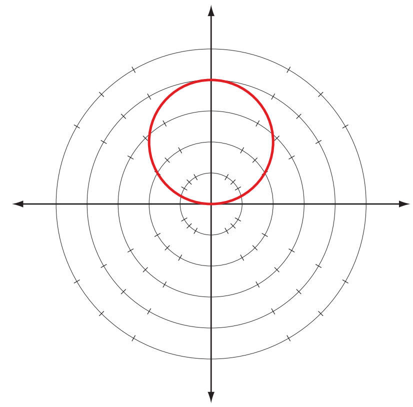
6. (a) $r^2 = 1^2 + (\sqrt{3})^2 \implies r = 2$. $\tan\theta = \sqrt{3}/1 \implies \theta = \pi/3$. The coordinates are $(2, \pi/3)$.

(b) $x = -2\cos(\pi/2) = 0$ and $y = -2\sin(\pi/2) = -2$, so the coordinates are $(0, -2)$.

7.

$$\begin{aligned} r^2 &= 4r\sin\theta \\ x^2 + y^2 &= 4y \\ x^2 + y^2 - 4y + 4 &= 4 \\ x^2 + (y-2)^2 &= 4 \end{aligned}$$

It is a circle, with center $(0, 2)$ and radius 2.



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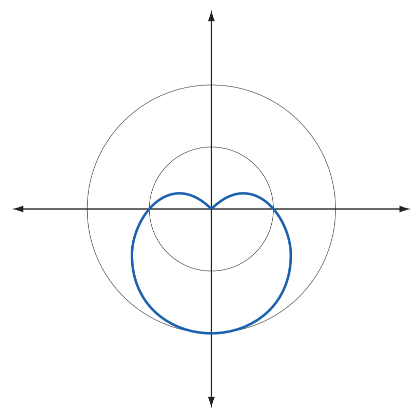

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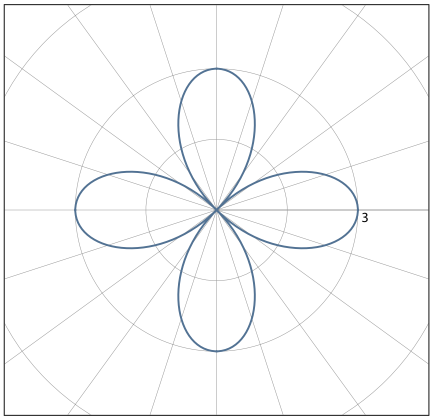
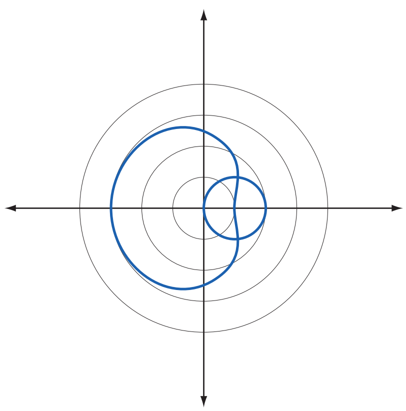


10. $2\cos(3\theta) = 0 \implies 3\theta = \pm\pi/2 \implies \theta = \pm\pi/6$, so one petal is traced out when $-\pi/6 \le \theta \le \pi/6$. Therefore

$$A = \int_{-\pi/6}^{\pi/6} \frac{1}{2}(2\cos(3\theta))^2 \, d\theta.$$

11.



The two curves intersect when $3\cos\theta = 2 - \cos\theta \implies \cos\theta = 1/2 \implies \theta = \pi/3$ (there is also an intersection in the fourth quadrant that this calculation does not pick up). Then

$$A = 2\int_0^{\pi/3} \frac{1}{2}((3\cos\theta)^2 - (2-\cos\theta)^2) \, d\theta.$$

12.

$$\begin{cases} x(\theta) = 1 + \sec^2\theta \\ y(\theta) = \tan\theta \end{cases}$$