MATH 2050 TEST 3. SPRING 2013

1. Use the method of *Integration by Parts* to compute the integral:

$$\int x^7 \ln(x)\, dx.$$

2. Use the method of *Trigonometric Substitution* to compute the integral:

$$\int \frac{x^2}{\sqrt{4-x^2}}\, dx.$$

3. Use the method of *Partial Fractions* to compute the integral:

$$\int \frac{2x^2+x+3}{x^3+3x^2}\, dx.$$

4. $\int \sin^3 x \cos^4 x\, dx$

5. $\int \tan x \sec^3 x\, dx$

6. $\int \frac{5x-7}{x^2-2x-3}\, dx$

7. $\int x^2 e^{4x}\, dx$

8. $\int \frac{x}{\sqrt{x^2+3}}\, dx$

9. $\int \frac{dx}{\sqrt{x^2+6x+8}}$

10. Evaluate the improper integral:

$$\int_{10}^{\infty} e^{-x}\, dx.$$

SOLUTIONS

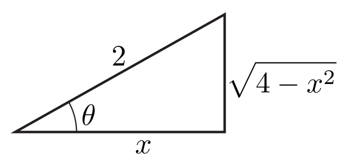
1. Use

$$u = \ln x \qquad dv = x^7\, dx$$
$$du = (1/x)\, dx \qquad v = x^8/8$$

to get

$$\int x^7 \ln x\, dx = \frac{1}{8}x^8 \ln x - \int \frac{1}{8}x^7\, dx = \frac{1}{8}x^8 \ln x - \frac{1}{64}x^8 + C.$$

2. Draw the triangle



$$x = 2\cos\theta$$
$$\sqrt{4-x^2} = 2\sin\theta$$
$$dx = -2\sin\theta\, d\theta$$

to get

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx$$
$$= \int \frac{4\cos^2\theta(-2\sin\theta d\theta)}{2\sin\theta}$$
$$= -4\int \cos^2\theta\,d\theta$$
$$= -4\int \frac{1}{2}(1+\cos(2\theta))\,d\theta$$
$$= -2\left[\theta + \frac{\sin(2\theta)}{2}\right] + C$$
$$= -2[\theta + \sin\theta\cos\theta] + C$$
$$= -2\left[\cos^{-1}(x/2) + x\sqrt{4-x^2}/4\right] + C$$

3. Partial fraction decomposition

$$\frac{2x^2+x+3}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

Matching up the numerators:

$$2x^2+x+3$$
$$= Ax(x+3) + B(x+3) + Cx^2$$
$$= Ax^2 + 3Ax + Bx + 3B + Cx^2$$
$$= (A+C)x^2 + (3A+B)x + 3B$$

Then $B = 1$, so $A = 0$, and so $C = 2$. That means

$$\int \frac{2x^2+x+3}{x^2(x+3)}\,dx$$
$$= \int \left(\frac{1}{x^2} + \frac{2}{x+3}\right)\,dx$$
$$= -\frac{1}{x} + 2\ln|x+3| + C$$

4.

$$\int \sin^3 x\cos^4 x\,dx$$
$$= \int (1-\cos^2 x)\cos^4 x\sin x\,dx$$

Put $u = \cos x$, so that $du = -\sin x\,dx$. Then

$$= \int (1-u^2)u^4(-du)$$
$$= \int u^6 - u^4\,du$$
$$= \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$$
$$= \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C$$

5.

$$\int \tan x\sec^3 x\,dx$$
$$= \int \sec^2 x\sec x\tan x\,dx$$

Put $u = \sec x$ so that $du = \sec x\tan x\,dx$:

$$= \int u^2\,du$$
$$= u^3/3 + C$$
$$= \sec^3 x/3 + C$$

6. Partial fractions.

$$\frac{5x-7}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

Matching up the numerators:

$$5x - 7 = A(x+1) + B(x-3)$$

Plug in $x = -1$ to get $B = 3$ and plug in $x = 3$ to get that $A = 2$. Then

$$\int \frac{5x-7}{(x-3)(x+1)}\,dx$$
$$= \int \left(\frac{2}{x-3} + \frac{3}{x+1}\right)\,dx$$
$$= 2\ln|x-3| + 3\ln|x+1| + C$$

7. Integration by parts

$u = x^2$	$dv = e^{4x}\,dx$
$du = 2x\,dx$	$v = e^{4x}/4$

$$\int x^2 e^{4x}\,dx$$
$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int xe^{4x}\,dx$$

Integration by parts again.

$$u = x \qquad dv = e^{4x}\,dx$$
$$du = dx \qquad v = e^{4x}/4$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left[\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,dx\right]$$
$$= \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$$

8. Substitution: put $u = x^2 + 3$ so that $du = 2xdx$. Then

$$\int \frac{x}{\sqrt{x^2+3}}\,dx$$
$$= \int \frac{(1/2)\,du}{\sqrt{u}}$$
$$= \frac{1}{2}\int u^{-1/2}\,du$$
$$= u^{1/2} + C$$
$$= \sqrt{x^2+3} + C$$

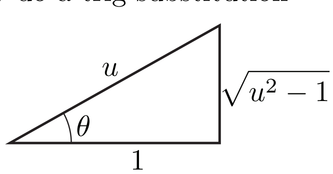
9. Your need to complete the square:

$$\int \frac{dx}{\sqrt{x^2+6x+8}} = \int \frac{dx}{\sqrt{(x+3)^2-1}}$$

Put $u = x + 3$ so that $du = dx$:

$$= \int \frac{du}{\sqrt{u^2-1}}$$

Now do a trig substitution



$$u = \sec\theta$$
$$\sqrt{u^2-1} = \tan\theta$$
$$du = \sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sec\theta\tan\theta d\theta}{\tan\theta}$$
$$= \int \sec\theta\,d\theta$$
$$= \ln|\sec\theta + \tan\theta| + C$$
$$= \ln\left|u + \sqrt{u^2-1}\right| + C$$
$$= \ln\left|(x+3) + \sqrt{(x+3)^2-1}\right| + C$$

10.

$$\int_{10}^{\infty} e^{-x}\,dx$$
$$= \lim_{a\to\infty}\int_{10}^{a} e^{-x}\,dx$$
$$= \lim_{a\to\infty}(-e^{-x})\Big|_{10}^{a}$$
$$= \lim_{a\to\infty}(-e^{-a} + e^{-10})$$
$$= e^{-10}$$