MATH 3060. FINAL EXAM (HARVEY FALL 2010).

Name (5 points):

No books, notes, or calculators are allowed. Please show all of your work. Each problem is worth 5 points.

1 Compute the limit of each sequence:

(*a*) $\left\{\dfrac{2n^2+3n+1}{3n^2+1}\right\}_{n=1}^{\infty}$

(*b*) $\left\{\dfrac{1}{2^n}\right\}_{n=0}^{\infty}$

2 Evaluate the following convergent series:

$$\sum_{n=0}^{\infty} \frac{2^{n+2}}{3^{n+1}}.$$

3 Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}.$$

4 Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n^3}{3^n}.$$

5 Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{(2n)!}.$$

6 Determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3 + 2n}.$$

7 Does the following alternating series converge or diverge? If it does converge, does it converge absolutely or conditionally?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/2}+1}.$$

8 State the interval of convergence of the following power series. Be sure to test the endpoints.

$$\sum_{n=1}^{\infty} \frac{1}{5^n\sqrt{n}}\, x^n.$$

9 What is the Taylor series, expanded about the point $c = 0$, for the function $f(x) = x^3 \sin(2x)$?

10 Use a Taylor series to estimate the value of $\cos(1/10)$ to an accuracy of $1/1000$ (be sure to justify the number of terms that you use in your approximation).

11 Find a parametric equation for the line which passes through the points $(1, 3, 5)$ and $(0, 4, -6)$.

12 Find the equation of the plane which passes through the points $(1, 1, 1)$, $(2, 4, 1)$ and $(3, 1, -5)$.

13 Sketch the quadratic surface given by the equation

$$x^2 - 2x + 4y^2 - 16y + z^2 = -13.$$

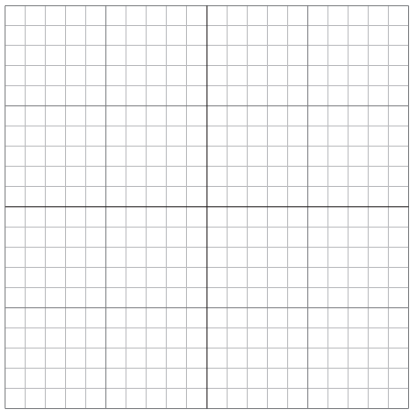
14 Compute the arc length of the parametric curve $r(t) = \langle 2\sin t, 3t, 2\cos t\rangle$, for $0 \leq t \leq 1$.

15 Compute the unit tangent $T(t)$ and the unit normal $N(t)$ when $t = 0$ for the parametric curve

$$r(t) = \langle 3\cos t, 5\sin t, 4\cos t \rangle.$$

16 Sketch the domain of the function of two variables

$$f(x, y) = \sqrt{x + y} + \sqrt{x - y}.$$



17 Let $f(x, y, z) = x^2yz + e^{xyz}$. Compute ∇f.

18 Find the equation of the tangent plane to the surface $z = x^2y + x$ at the point $(1, 2, 3)$.

19 Compute the directional derivative to the function $f(x, y) = x^2 - xy + y^2$ in the direction $\langle 1, -1 \rangle$ at the point $(1, 2)$.

20 Find the absolute maximum and minimum values of $f(x, y) = 2x^3 + y^4$ on the closed unit disk $D = \{(x, y)|x^2 + y^2 \leq 1\}$.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

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