Math 306. Final Exam (Harvey Spring 2009).

Name (4 points):

No books, notes, or calculators are allowed. Please show all of your work. Each problem is worth 4 points.

1 Compute the limit of the following sequence

$$\left\{ \frac{n^2 + n \ln n}{3n^2 + 4n} \right\}_{n=1}^{\infty}$$

2 Does the following series converge or diverge?

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

3 Does the following series converge or diverge?

$$\sum_{n=0}^{\infty} \frac{n+1}{n^3+4n+2}$$

4 Does the following series converge or diverge?

$$\sum_{n=0}^{\infty} \frac{n!(n+1)!}{(2n)!}$$

5 Does the following alternating series converge? If so, does it converge conditionally or absolutely?

$$\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$$

6 The series below converges. Determine what it converges to.

$$\sum_{n=1}^{\infty} \frac{3^{n-1}}{4^n}$$

7 Use the definition of the Taylor series as the sum

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

to *directly* calculate the Taylor series expanded about the point $c = 0$ for the function $f(x) = \sinh x$. Do *not* derive this series from a known series. Hints: $(\sinh x)' = \cosh x$ and $(\cosh x)' = \sinh x$; $\sinh 0 = 0$ and $\cosh 0 = 1$.

8 Find the interval of convergence of the following power series. Do not forget to check the behavior at any endpoints.

$$\sum_{n=1}^{\infty} \frac{x^n}{3^n \cdot n^3}$$

9 Use a known series to find the Taylor series expanded about the point $c = 0$ for the function $f(x) = x\cos(2x^2)$.

10 Use a Taylor series to estimate the value of $\cos(1)$ to within a margin of error of $1/100$.

11 Find a series solution to the integral

$$\int_0^5 e^{-x^2}\, dx$$

12 Find the equation of the line which passes through the point $(2, 0, 1)$ and is parallel to the line given by the parametric equations $r(t) = \langle 1 + 4t, 2 - 3t, 1 + 2t \rangle$.

13 Find the equation of the plane which passes through the three points $(1, 1, 1)$, $(2, 4, 1)$ and $(3, 1, 0)$.

14 Sketch the quadric surface

$$x^2 + 4y^2 + 9z^2 = 36.$$

15 Let $v = \langle 1, 2, 3 \rangle$ and $w = \langle -3, 1, 2 \rangle$. Compute $v \cdot w$ and $v \times w$.

16 Let $r(t) = \langle \sin 4t, t, \cos 4t \rangle$. Compute the unit tangent $T(0)$ and the unit normal $N(0)$.

17 Compute the arc length of the curve

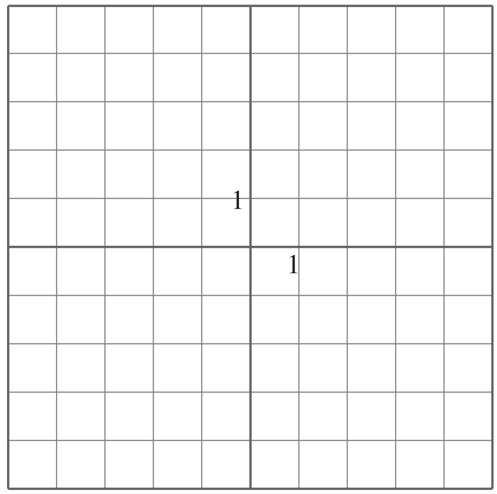
$$r(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$$

between $t = 0$ and $t = 1$.

18 Find the radius and the coordinates of the center of the sphere given by the equation

$$x^2 - 2x + y^2 - 6y + z^2 + 4z = -5$$

19 Sketch the level curves $f(x, y) = c$ of the function $f(x, y) = x - y^2$ corresponding to the values $c = -2$, $c = 0$, and $c = 2$.



20 Verify that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + y^2}{x^2 + y^2}$$

21 Find the equation of the tangent plane to the surface

$$xy^2z + z^3 - 2x^2 = 0$$

at the point $(1,1,1)$.

22 Find the critical point(s) of the function

$$f(x,y) = x - y^2 - \ln(x+y).$$

23 Find three positive numbers x, y, and z whose sum is 18 and whose product is a maximum.

24 Compute the directional derivative of the function $f(x, y) = x^2e^{-y}$ at the point $(2, 0)$ in the direction $\langle 1, 3 \rangle$.

25 Let $R = \{(x,y) \big| 0 \le x \le 4,\, 0 \le y \le 1\}$. Compute

$$\iint_R x + 3y^2 \, dA.$$

Pledge: