Math 3060 Test 3. Fall 2010

1 (10 points) Let $\mathbf{v} = \langle 1, 3, -5\rangle$ and $\mathbf{w} = \langle 2, 1, -1\rangle$. (a) Compute $3\mathbf{v} - 4\mathbf{w}$. (b) Find a unit vector in the same direction as $\mathbf{v}$.

2 (5 points) Consider the two vectors $\mathbf{v} = \langle 3, x, 1\rangle$ and $\mathbf{w} = \langle 2, 1, -1\rangle$. What value(s) of x will make these two vectors perpendicular?

3 (10 points) Let $\mathbf{v} = \langle 1, 3, -5\rangle$ and $\mathbf{w} = \langle 2, 1, -1\rangle$. (a) Compute $\mathbf{v} \cdot \mathbf{w}$. (b) Compute $\mathbf{v} \times \mathbf{w}$.

4 (10 points) Find parametric equations for the line through the points $(2, 1, 4)$ and $(-3, 4, 2)$.

5 (10 points) Find the equation of the plane through points $(1, 1, 2)$, $(2, 0, 3)$ and $(3, 1, -4)$.

6 (10 points) Give parametric equations for the intersection line of the two planes

$$x + y + z = 5 \quad \& \quad x - y + 2z = 3.$$

7 (10 points) Sketch the graph of the quadratic surface $4x^2 + y^2 = 4$.

8 (10 points) Compute

$$\lim_{t\to 0} \left\langle \frac{2t}{1+t}, \frac{\sin(2t)}{3t}, e^{5t} \right\rangle.$$

9 (10 points) Find the arc length of

$$\mathbf{r}(t) = \left\langle 5, t, \frac{1}{3}t^3 + \frac{1}{4t} \right\rangle, \quad 1 \le t \le 2.$$

10 (15 points) Let

$$\mathbf{r}(t) = \langle 1 + 3\cos(2t), t, 3\sin(2t)\rangle.$$

(a) Compute $T(0)$.

(b) Compute $N(0)$.

(c) Compute $B(0)$.

SOLUTIONS

1

$$a)\quad 3\mathbf{v}-4\mathbf{w} = 3\langle 1,3,-5\rangle - 4\langle 2,1,-1\rangle = \langle -5,5,-11\rangle$$

$$b)\quad |\mathbf{v}| = \sqrt{35} \implies \frac{\mathbf{v}}{|\mathbf{v}|} = \left\langle \frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, -\frac{5}{\sqrt{35}} \right\rangle.$$

2 We need $\mathbf{v}\cdot\mathbf{w}=0$:

$$\begin{aligned} \langle 3,x,1\rangle \cdot \langle 2,1,-1\rangle &= 0 \\ 6+x-1 &= 0 \\ x &= -5 \end{aligned}$$

3

$$a)\quad \mathbf{v}\cdot\mathbf{w} = \langle 1,3,-5\rangle \cdot \langle 2,1,-1\rangle = 10$$

$$b)\quad \mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -5 \\ 3 & 1 & -1 \end{vmatrix} = \langle 2,-9,-5\rangle$$

4

$$\begin{aligned} \mathbf{r_0} &= \langle 2,1,4\rangle \\ \mathbf{d} &= \langle -5,3,-2\rangle \\ \mathbf{r}(t) &= r_0 + td \\ &= \langle 2,1,4\rangle + t\langle -5,3,-2\rangle \\ &= \langle 2-5t, 1+3t, 4-2t\rangle \end{aligned}$$

5

$$\begin{aligned} \mathbf{v_1} &= \langle 1,-1,1\rangle \\ \mathbf{v_2} &= \langle 2,0,-6\rangle \\ \mathbf{n} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 2 & 0 & -6 \end{vmatrix} = 6\mathbf{i}+8\mathbf{j}+2\mathbf{k} \end{aligned}$$

The equation of the plane is

$$\begin{aligned} \langle 6,8,2\rangle \cdot \langle x-1, y-1, z-2\rangle &= 0 \\ 6x+8y+2z &= 18 \\ 3x+4y+z &= 9 \end{aligned}$$

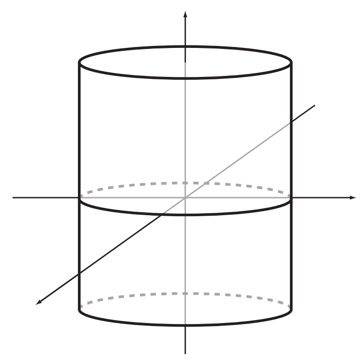
6 Add the two equations to get $2x+3z=8$. Therefore, if we put $x=t$, then $z=(8-2t)/3$. Now

$$\begin{aligned} y &= 5-x-z \\ &= 5-t-((8-2t)/3) \\ &= (7/3)-(t/3) \end{aligned}$$

so

$$r(t) = \left\langle t, \frac{7}{3}-\frac{1}{3}t, \frac{8}{3}-\frac{2}{3}t \right\rangle.$$

7



8

$$\lim_{t\to 0} \left\langle \frac{2t}{1+t}, \frac{\sin(2t)}{3t}, e^{5t} \right\rangle = \left\langle 0, \frac{2}{3}, 1 \right\rangle.$$

9

$$
\begin{aligned}
r'(t) &= \left\langle 0, 1, t^2 - \frac{1}{4t^2} \right\rangle \\
|r'(t)| &= \sqrt{1 + \left(t^2 - \frac{1}{4t^2}\right)^2} \\
&= \sqrt{1 + t^4 - \frac{1}{2} + \frac{1}{16t^4}} \\
&= \sqrt{\left(t^2 + \frac{1}{4t^2}\right)^2} \\
&= t^2 + \frac{1}{4}t^{-2}
\end{aligned}
$$

$$
\begin{aligned}
s &= \int_1^2 t^2 + \frac{1}{4}t^{-2}\,dt \\
&= \left(\frac{t^3}{3} - \frac{1}{4}t^{-1}\right)\Bigg|_1^2 \\
&= 59/24
\end{aligned}
$$

10

$$
\begin{aligned}
r'(t) &= \langle -6\sin(2t), 1, 6\cos(2t) \rangle \\
|r'(t)| &= \sqrt{1 + 36\sin^2(2t) + 36\cos^2(2t)} = \sqrt{37} \\
T(t) &= \left\langle -\frac{6\sin(2t)}{\sqrt{37}}, \frac{1}{\sqrt{37}}, \frac{6\cos(2t)}{\sqrt{37}} \right\rangle \\
T(0) &= \left\langle 0, \frac{1}{\sqrt{37}}, \frac{6}{\sqrt{37}} \right\rangle.
\end{aligned}
$$

$$
\begin{aligned}
T'(t) &= \left\langle -\frac{12\cos(2t)}{\sqrt{37}}, 0, -\frac{12\sin(2t)}{\sqrt{37}} \right\rangle \\
T'(0) &= \left\langle -\frac{12}{37}, 0, 0 \right\rangle \\
N(0) &= \langle -1, 0, 0 \rangle
\end{aligned}
$$

$$
\begin{aligned}
B(0) &= T(0) \times N(0) \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1/\sqrt{37} & 6/\sqrt{37} \\ -1 & 0 & 0 \end{vmatrix} \\
&= \left\langle 0, -\frac{6}{\sqrt{37}}, \frac{1}{\sqrt{37}} \right\rangle
\end{aligned}
$$