Math 306 Test 3. Fall 2008

1 (8 points) Let $\mathbf{v} = \langle 2,3,1\rangle$ and $\mathbf{w} = \langle 3,-1,1\rangle$. (a) Compute $\mathbf{v}\cdot\mathbf{w}$. (b) Compute $\mathbf{v}\times\mathbf{w}$.

2 (5 points) Find the unit vector which is in the same direction as the vector $\langle -1,-2,-3\rangle$.

3 (6 points) Compute the angle between the vectors $\mathbf{v} = \langle 2,0,0\rangle$ and $\mathbf{w} = \langle -1,0,1\rangle$.

4 (8 points) Let $\mathbf{v} = \langle 2,1\rangle$ and $\mathbf{w} = \langle 3,4\rangle$. Compute $\text{comp}_{\mathbf{v}}(\mathbf{w})$ and $\text{proj}_{\mathbf{v}}(\mathbf{w})$.

5 (8 points) Find parametric equations for the line which passes through the points $(1,2,6)$ and $(2,0,-4)$.

6 (8 points) Find parametric equations for the line which passes through the point $(2,2,3)$ and is parallel to the line

$$r(t) = \langle 1+t, 2-t, 3+2t\rangle.$$

7 (10 points) Find the equation of the plane which passes through the points $(1,2,1)$, $(2,0,1)$, and $(1,0,0)$.

8 (10 points) Sketch the graphs of the surfaces $y+2z = 2$ and $z^2 = x^2+y^2$. Describe the projection of their intersection curve onto the xy–plane.

9 (9 points) Identify the following surfaces (either by name or with a sketch).

$$(a) \quad 4x^2+y^2+9z^2 = 1$$

$$(b) \quad z = 9-(x^2+y^2)$$

$$(c) \quad 4x^2+z = 1$$

10 (10 points) Find the length of the curve

$$r(t) = \langle \sin(2t), \cos(2t), 8t\rangle$$

traced out between $t=0$ and $t=\pi$.

11 (15 points) Let

$$r(t) = \langle 4t, \sin(3t), -\cos(3t)\rangle$$

Compute the unit tangent, unit normal, and binormal vectors at the point $(0,0,-1)$.

1

$$\mathbf{v}\cdot\mathbf{w} = 4$$
$$\mathbf{v}\times\mathbf{w} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 3 & -1 & 1 \end{vmatrix} = \langle 4, 1, -11\rangle$$

2

$$|\langle -1,-2,-3\rangle| = \sqrt{14}$$
$$\implies \mathbf{u} = \langle -1/\sqrt{14}, -2/\sqrt{14}, -3/\sqrt{14}\rangle$$

3

$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{|\mathbf{v}|\cdot|\mathbf{w}|} = \frac{-2}{2\sqrt{2}} = -\sqrt{2}/2$$
$$\theta = \cos^{-1}(-\sqrt{2}/2) = 3\pi/4$$

4

$$comp_{\mathbf{v}}\mathbf{w} = \frac{\langle 2,1\rangle\cdot\langle 3,4\rangle}{|\langle 2,1\rangle|} = 2\sqrt{5}$$
$$proj_{\mathbf{v}}\mathbf{w} = \frac{\langle 2,1\rangle\cdot\langle 3,4\rangle}{|\langle 2,1\rangle|^2}\cdot\langle 2,1\rangle = \langle 4,2\rangle$$

5

$$r_0 = \langle 1,2,6\rangle$$
$$d = \langle 1,-2,-10\rangle$$
$$r(t) = r_0 + td = \langle 1+t, 2-2t, 6-10t\rangle$$

6

$$r_0 = \langle 2,2,3\rangle$$
$$d = \langle 1,-1,2\rangle$$
$$r(t) = r_0 + td = \langle 2+t, 2-t, 3+2t\rangle$$

7 First we find two vectors in the plane.

$$v_1 = \langle 1,-2,0\rangle$$
$$v_2 = \langle 0,-2,-1\rangle$$

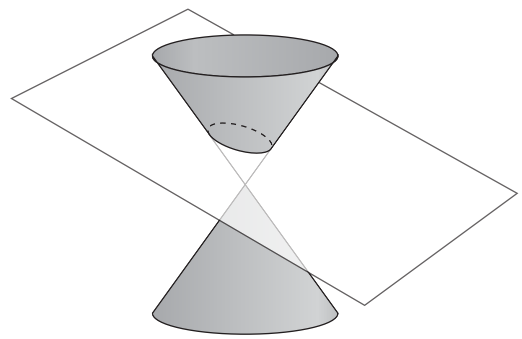
Then

$$n = \begin{vmatrix} i & j & k \\ 1 & -2 & 0 \\ 0 & -2 & -1 \end{vmatrix} = \langle -2,1,-2\rangle$$

so the equation of the plane is

$$\langle -2,1,-2\rangle\cdot\langle x-1, y, z\rangle = 0$$
$$-2x + y - 2z = -2$$

8



The projection of the intersection is an ellipse.

9 (a) ellipsoid. (b) paraboloid. (c) parabolic cylinder

10 First compute

$$r'(t) = \langle 2\cos(2t), -2\sin(2t), 8\rangle.$$

Then

$$\begin{aligned} L &= \int_0^\pi |r'(t)|\,dt \\ &= \int_0^\pi \sqrt{4\cos^2(2t)+4\sin^2(2t)+64}\,dt \\ &= \int_0^\pi \sqrt{68}\,dt \\ &= \sqrt{68}\pi. \end{aligned}$$

11

$$r'(t) = \langle 4, 3\cos(3t), -3\sin(3t)\rangle$$
$$\implies |r'(t)| = \sqrt{16 + 9\cos^2(3t) + 9\sin^2(3t)} = 5$$

$$T(t) = \left\langle \frac{4}{5}, \frac{3}{5}\cos(3t), -\frac{3}{5}\sin(3t)\right\rangle$$
$$\implies T(0) = \langle 4/5, 3/5, 0\rangle$$

$$T'(t) = \langle -\frac{9}{5}\sin(3t), -\frac{9}{5}\cos(3t)\rangle$$
$$\implies N(t) = \langle 0, -\sin(3t), -\cos(3t)\rangle$$
$$\implies N(0) = \langle 0, 0, -1\rangle$$

$$B(0) = \begin{vmatrix} i & j & k \\ 4/5 & 3/5 & -3/5 \\ 0 & 0 & -1 \end{vmatrix} = \langle -3/5, 4/5, 0\rangle$$