Math 306 Test 3. Spring 2009

1 Compute the length of the vector $\mathbf{v} = \langle 3, 1, 4 \rangle$.

2 Find the angle between the vectors $\langle 1, 1, 0 \rangle$ and $\langle 0, 1, 1 \rangle$.

3 Let $\mathbf{v} = \langle 1, 3, -2 \rangle$ and $\mathbf{w} = \langle 2, 1, 1 \rangle$. (a) Compute $\mathbf{v} \cdot \mathbf{w}$. (b) Compute $\mathbf{v} \times \mathbf{w}$.

4 Let $\mathbf{v} = \langle 3, 1 \rangle$ and $\mathbf{w} = \langle 2, 2 \rangle$. What are $\text{comp}_{\mathbf{v}}\mathbf{w}$ and $\text{proj}_{\mathbf{v}}\mathbf{w}$?

5 Find parametric equations for the line through the points $(2, 1, 5)$ and $(3, 1, 3)$.

6 Find the equation of the plane through the points $(1, 2, 4)$, $(3, 1, -2)$ and $(2, 1, 0)$.

7 Sketch the following quadric surface:

$$z = x^2 + y^2 - 2.$$

8 Sketch the surface

$$z^2 = x^2 + y^2.$$

9 Compute the length of the curve $r(t) = \left\langle t, \frac{2}{3}t^{3/2}, 3 \right\rangle$ when $1 \le t \le 8$.

10-12 For these problems, we consider the vector function $r(t) = \langle \sin(t^2), \cos(t^2), 3 \rangle$.

10 Compute the unit tangent vector $T(t)$. What is $T(0)$?

11 Compute the unit normal vector $N(t)$. What is $N(0)$?

12 Compute $B(0)$, the binormal vector when $t = 0$.

Solutions

1

$$|\mathbf{v}| = \sqrt{9 + 1 + 16} = \sqrt{26}.$$

2

$$\begin{aligned} \cos\theta &= \frac{\langle 1, 1, 0 \rangle \cdot \langle 0, 1, 1 \rangle}{|\langle 1, 1, 0 \rangle| \cdot |\langle 0, 1, 1 \rangle|} \\ &= \frac{1}{\sqrt{2} \cdot \sqrt{2}} = 1/2 \\ \implies \theta &= \pi/3 \end{aligned}$$

3

$$\mathbf{v} \cdot \mathbf{w} = 2 + 3 - 5 = 3$$

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -2 \\ 2 & 1 & 1 \end{vmatrix} = \langle 5, -5, -5 \rangle$$

4

$$comp_{\mathbf{v}}\mathbf{w} = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}|} = \frac{6 + 2}{\sqrt{10}} = \frac{8}{\sqrt{10}}$$

$$proj_{\mathbf{v}}\mathbf{w} = \frac{\mathbf{v} \cdot \mathbf{w}}{\mathbf{v} \cdot \mathbf{v}} \cdot \mathbf{v} = \frac{8}{10}\langle 3, 1 \rangle = \left\langle \frac{12}{5}, \frac{4}{5} \right\rangle$$

5 The direction vector is $\langle 1, 0, -2 \rangle$, so the vector equation of the line is

$$r(t) = \langle 2, 1, 5 \rangle + t\langle 1, 0, -2 \rangle$$

In parametric form

$$\begin{aligned} x(t) &= 2 + t \\ y(t) &= 1 \\ z(t) &= 5 - 2t \end{aligned}$$

6 Compute two vectors which lie in the plane by taking differences of coordinates of pairs of points:

$$\mathbf{v_1} = \langle 2, -1, -6 \rangle$$
$$\mathbf{v_2} = \langle 1, -1, -4 \rangle$$

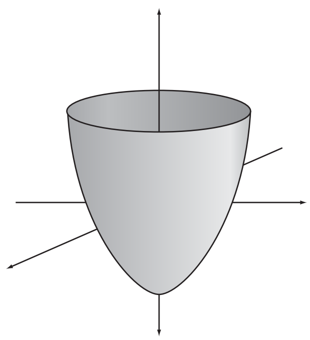
The normal vector is

$$\mathbf{n} = \mathbf{v_1} \times \mathbf{v_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -6 \\ 1 & -1 & -4 \end{vmatrix} = \langle -2, 2, -1 \rangle.$$

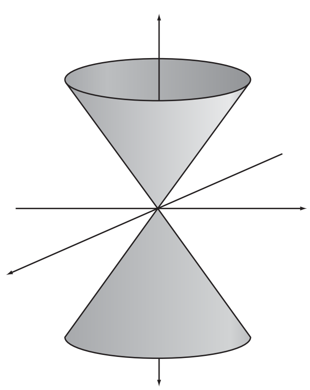
The equation of the plane is

$$\langle -2, 2, -1 \rangle \cdot \langle x-1, y-2, z-4 \rangle = 0$$
$$-2(x-1) + 2(y-2) - (z-4) = 0$$
$$2x - 2y + z = 2$$

7



8



9 First calculate the length of $r'(t)$

$$r'(t) = \langle 1, t^{1/2}, 0 \rangle$$
$$\implies \sqrt{1+t}$$

Therefore the arc length of the curve is

$$\begin{aligned} L &= \int_1^8 \sqrt{1+t}\, dt \\ &= \frac{2}{3}(1+t)^{3/2}\Big|_1^8 \\ &= 18 - \frac{2}{3}\sqrt{8} \end{aligned}$$

10

$$r'(t) = \langle 2t\cos(t^2), -2t\sin(t^2), 0 \rangle$$
$$|r'(t)| = \sqrt{4t^2\cos^2(t^2) + 4t^2\sin^2(t^2)} = 2t$$

$$T(t) = \langle \cos(t^2), -\sin(t^2), 0 \rangle$$
$$T(0) = \langle 1, 0, 0 \rangle$$

11

$$T'(t) = \langle -2t\sin(t^2), -2t\cos(t^2), 0 \rangle$$
$$|T'(t)| = \sqrt{4t^2\sin^2(t^2) + 4t^2\cos^2(t^2)} = 2t$$

$$N(t) = \langle -\sin(t^2), -\cos(t^2), 0 \rangle$$
$$N(0) = \langle 0, -1, 0 \rangle$$

12

$$\begin{aligned} B(0) &= T(0) \times N(0) \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \\ &= \langle 0, 0, -1 \rangle. \end{aligned}$$