

## IN-CLASS PRACTICE FOR TEST 3

**1** Compute:

$$\langle 2, 1, 5 \rangle \cdot \langle 3, 1, 1 \rangle$$

**2** Compute

$$\langle 2, 1, 5 \rangle \times \langle 3, 1, 1 \rangle$$

**3** Find the angle between the vectors:

$$v_1 = \langle 3, 1, 2 \rangle \quad v_2 = \langle 2, 1, 0 \rangle$$

**4** Find the unit vector in the same direction as

$$v = \langle 4, 2, 2 \rangle$$

**5** Calculate the projection of  $v$  onto  $w$ :

$$v = \langle 3, 1, 2 \rangle \quad w = \langle 2, 1, -1 \rangle$$

**6** Find the equation of the line which passes through the points  $(3, 0, 2)$  and  $(1, 1, 4)$ .

**7** Find the equation of the plane which passes through the points

$$(2, 1, 5) \quad (3, 1, 3) \quad (1, 2, 2)$$

**8** Find the equation of the line which passes through the point  $(1, 2, 0)$  and is perpendicular to both  $\langle 1, 1, 0 \rangle$  and  $\langle 0, 1, 1 \rangle$ .

**9** The line

$$r(t) = \langle 1 + t, 2 - t, 3 + 2t \rangle$$

and the plane

$$x - 2y + z = 10$$

intersect at a point. Find the coordinates of the point.

**10** Sketch the graph of the following surface:

$$z = x^2 + y^2$$

Problems 1 – 5 involve the space curve given by the parametric equation:

$$R(t) = \langle \cos t, \sin t, t \rangle$$

**11** Compute  $R'(t)$ .

**12** Compute the unit tangent vector  $T(t)$ .

**13** Compute the unit normal vector  $N(t)$ .

**14** Find  $T(0)$  and  $N(0)$ .

**15** Compute the binormal vector  $B(0)$ .

**16** Find the curvature of the plane curve  $R(t) = \langle t + 1, t^3 + 1 \rangle$  at the point  $(1, 1)$ . You might be able to use one of the following formulas for the curvature of a plane curve:

$$k = \frac{|x''y' - x'y''|}{[(x')^2 + (y')^2]^{3/2}}$$

$$k = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$$

SOLUTIONS

1

$$6 + 1 + 5 = 12$$

2

$$\begin{vmatrix} i & j & k \\ 2 & 1 & 5 \\ 3 & 1 & 1 \end{vmatrix} = -4i + 13j - k$$

3

$$\cos \theta = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|} = \frac{7}{\sqrt{14}\sqrt{5}}$$

$$\theta = \cos^{-1} \left( \frac{7}{\sqrt{70}} \right)$$

4

$$\|v\| = \sqrt{24}$$

$$\begin{aligned} \Rightarrow u &= \left\langle \frac{4}{\sqrt{24}}, \frac{2}{\sqrt{24}}, \frac{2}{\sqrt{24}} \right\rangle \\ &= \left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle \end{aligned}$$

5

$$\begin{aligned} \text{proj}_w(v) &= \frac{v \cdot w}{w \cdot w} w \\ &= \frac{5}{6} \langle 2, 1, -1 \rangle \\ &= \left\langle \frac{5}{3}, \frac{5}{6}, -\frac{5}{6} \right\rangle \end{aligned}$$

6

$$\begin{aligned} r_0 &= \langle 3, 0, 2 \rangle \\ d &= \langle -2, 1, 2 \rangle \end{aligned}$$

$$\begin{aligned} \Rightarrow r(t) &= \langle 3, 0, 2 \rangle + t \langle -2, 1, 2 \rangle \\ &= \langle 3 - 2t, t, 2 + 2t \rangle \end{aligned}$$

7 Construct two vectors in the plane:

$$v_1 = \langle 1, 0, -2 \rangle \quad v_2 = \langle -1, 1, -3 \rangle$$

The normal vector to the plane is their cross product:

$$n = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ -1 & 1 & -3 \end{vmatrix} = 2i + 5j + k$$

The equation of the plane is:

$$\begin{aligned} n \cdot \langle x - 2, y - 1, z - 5 \rangle &= 0 \\ 2(x - 2) + 5(y - 1) + (z - 5) &= 0 \\ 2x + 5y + z &= 14 \end{aligned}$$

8 Since the line is perpendicular to the two vectors, we can find its direction vector by taking the cross product of the two vectors:

$$d = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = i - j + k$$

The equation of the line is

$$\begin{aligned} r(t) &= \langle 1, 2, 0 \rangle + t \langle 1, -1, 1 \rangle \\ &= \langle 1 + t, 2 - t, t \rangle \end{aligned}$$

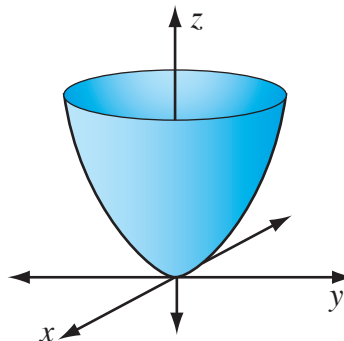
9 To find the “time” of intersection, plug in:

$$(1 + t) - 2(2 - t) + (3 + 2t) = 10 \implies t = 2$$

To find the point of intersection:

$$r(2) = \langle 3, 0, -1 \rangle$$

10



11

$$R'(t) = \langle -\sin t, \cos t, 1 \rangle$$

12

$$\|R'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$T(t) = \left\langle -\frac{\sin t}{\sqrt{2}}, \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

13

$$T'(t) = \left\langle -\frac{\cos t}{\sqrt{2}}, -\frac{\sin t}{\sqrt{2}}, 0 \right\rangle$$

$$\|T'(t)\| = 1/\sqrt{2}$$

$$N(t) = \langle -\cos t, -\sin t, 0 \rangle$$

14

$$T(0) = \langle 0, 1/\sqrt{2}, 1/\sqrt{2} \rangle$$

$$N(0) = \langle -1, 0, 0 \rangle$$

15

$$B(0) = T(0) \times N(0)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ -1 & 0 & 0 \end{vmatrix}$$

$$= \langle 0, -1/\sqrt{2}, 1/\sqrt{2} \rangle$$

16

$$x = t + 1 \implies x' = 1 \implies x'' = 0$$

$$y = t^3 + 1 \implies y' = 3t^2 \implies y'' = 6t$$

$$k = \frac{|6t|}{[1 + 9t^4]^{3/2}}$$

Note that the curve passes through the point (1, 1) when  $t = 0$ :

$$k(0) = 0$$