# Math 3350. Test 1 (Harvey Spring 2012).

Name: ______________________________________________

[No books or notes are allowed. Please show all of your work.]

1. (6 points) A set $S$ with binary operation $\circ$ is a *group* if:

2. (6 points) The 2-step subgroup test: A nonempty subset $H$ of a group $G$ is a *subgroup* if:

3. (6 points) Congruence mod $n$: $a \equiv b \mod n$ if:

4. (6 points) A group $G$ is *abelian* if:

5. (6 points) The *order* of a group $G$ is:

6. (8 points) List the elements in the subgroup generated by 10 in $\mathbb{Z}_{16}$.

7. (8 points) Give an example of a finite non-abelian group. Give an example of an infinite abelian group.

8. (8 points) Give an example of a finite subgroup $H$ of an infinite group $G$ (other than the trivial example, where $H = e$).

9. (8 points) What is the inverse of the element 7 in $U(10)$?

10. (8 points) Construct the Cayley table for the unitary group $U(15)$. Is $U(15)$ cyclic? Why or why not?

11. (10 points) The group $M(2,\mathbb{R})$ consists of the set of all $2 \times 2$ matrices whose entries are real numbers together with the operation of matrix addition. Let $H$ be the set of all *symmetric* matrices in $M(2,\mathbb{R})$– that is,

$$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M(2,\mathbb{R}) \,\middle|\, b = c \right\}.$$

Prove that $H$ is a subgroup of $M(2,\mathbb{R})$.

12. (10 points) Suppose that $a^3 \neq e$ and $a^7 = e$. Show that $a^5 \neq e$.

13. (10 points) Let $k$ be a fixed, positive integer, and let $G$ be an abelian group. Define the $k^{th}$ powers of $G$, written $G^k$, to be the set of elements in $G$ that can be written in the form $g^k$ for some $g \in G$. That is,

$$G^k = \{h \in G \mid h = g^k \text{ for some } g \in G\ \}.$$

Prove that $G^k$ is a subgroup of $G$. Give two examples (with $k > 1$): one where $G^k = G$, and one where $G^k \neq G$.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

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