MATH 3350. TEST 1 (HARVEY SPRING 2013).

Name (4 points): ____________________

[No books, notes, or calculators are allowed. Please show all of your work.]

Definitions. Be precise and concise. Do not give vague answers.

1. (6 points) Suppose that H is a nonempty subset of a group G. What are the two steps of the 2-Step Subgroup Test (to show that H is a subgroup of G)?

2. (4 points) What does it mean for a group G to be *abelian*?

3. (6 points) What is the *order* of a group G? What is the *order* of an element a in a group G?

4. (6 points) What is the *center* $Z(G)$ of a group G? What does it mean if $G = Z(G)$?

5. (6 points) Describe the elements of the group $U(n)$. What is the operation in this group?

Problems. Short answers are sufficient.

6. (4 points) Is $U(n)$ a subgroup of $\mathbb{Z}_n$? Why or why not?

7. (10 points) Identify each group in the list below as finite or infinite, and abelian or non-abelian (put checks in the appropriate boxes).

Group	Finite	Infinite	Abelian	non-Abelian
$\mathbb{Z}_8$	☐	☐	☐	☐
D_8	☐	☐	☐	☐
$U(8)$	☐	☐	☐	☐
$\mathbb{Z}$	☐	☐	☐	☐
$\mathbb{R}^{\times}$	☐	☐	☐	☐

8. (6 points) What is the order of the group $\mathbb{Z}_{10}$? What is the order of the group $U(10)$?

9. (6 points) What is the order of the element 2 in $U(15)$? What is the order of the element 2 in $\mathbb{Z}_{15}$?

10. (6 points) Construct the Cayley table for the group $U(6)$.

11. (6 points) What is the inverse of the element 3 in the group $\mathbb{Z}_{10}$? What is the inverse of the element 3 in the group $U(10)$?

Proofs. Give clear arguments. Don't skip steps.

12. (10 points) Let m and n be positive integers, and suppose that m divides n. Let H be the subset of $\mathbb{Z}_n$ consisting of all elements which are a multiple of m. [For instance, if $m = 3$ and $n = 12$, then $H = \{0, 3, 6, 9\} \subseteq \mathbb{Z}_{12}$.] Prove that H is a subgroup of Z_n.

13. (10 points) Let a and b be elements in an abelian group G with identity e. Suppose that $a^9 = e$, $b^2 = e$, and $(ab)^6 = e$. Prove that $a^2 = a^{-1}$.

14. (10 points) Let x be an element of G, and let H be a subgroup of G. Let a and b be integers, and let $g = gcd(a, b)$. Prove that if both $x^a \in H$ and $x^b \in H$, then $x^g \in H$.