MATH 3350. TEST 2 (HARVEY SPRING 2012).

Name (5 points): ______________________________

[No books or notes are allowed. Please show all of your work.]

1. (6 points) Definition: A group $G$ is *cyclic* if ...

2. (6 points) Definition: If $g$ is an element of a group $G$, then the *order* of $g$ is ...

3. (6 points) If $G$ is a cyclic group with order $n$, what is the connection between the subgroups of $G$ and the number $n$?

4. (10 points) Multiply the permutations. Write your answer as a product of disjoint cycles.

   (a) $(135)(246) \cdot (256)(13)$

   (b) $(12)(3578) \cdot (26)(35)(57)$

5. (8 points) Draw the subgroup lattice of the group $\mathbb{Z}_{45}$.

6. (6 points) Give an example of an abelian group with exactly 24 elements. Give an example of a non-abelian group with exactly 24 elements.

7. (12 points) What are the orders of the following elements in their groups?

   (a) 4 in $U(7)$

   (b) 12 in $\mathbb{Z}_{20}$

   (c) (132) in $S_3$

   (d) (13)(2468)(579) in $S_9$

8. (6 points) Give an example of two *different* elements in a cyclic group that generate the *same* subgroup.

9. (6 points) Let $H$ be the subgroup of order 6 in $\mathbb{Z}_{30}$. List all the elements of $H$. List all the generators of $H$.

10. (10 points) Prove that $U(20)$ is not cyclic.

11. (10 points) Consider the group $\mathbb{Q}$ (with $+$) and a subset $H$ of it consisting of all rational numbers that can be written in the form $a/3^b$, where $a, b$ are integers. That is,

$$H = \left\{ \frac{a}{3^b} \in \mathbb{Q} \,\middle|\, a, b \in \mathbb{Z} \right\}.$$

Prove that $H$ is a subgroup of $\mathbb{Q}$.

12. (10 points) In a group $G$, let $a$ and $b$ be elements with finite order, say $|a| = m$ and $|b| = n$. Prove that if $a \cdot b$ has infinite order, then $G$ is non-abelian.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

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