MATH 3350. TEST 2 (HARVEY SPRING 2013).

Name (3 points): ______________________________

[No books, notes, or calculators are allowed. Please show all of your work.]

Definitions. Be precise and concise. Do not give vague answers.

1. (5 points) A group G is *cyclic* if:

2. (5 points) A map $f : G \to H$ is an *isomorphism* if:

3. (5 points) An *inner automorphism* of a group G is:

Problems. Short answers are sufficient. ______________________________

4. (3 points) What is the inverse of the permutation (12345)?

5. (6 points) Give the order of each of the following groups:

- $U(5)$

- S_5

- A_5

6. (6 points) Give the order of each of the following permutations:

- $(123)(45)$

- $(13)(12)$

- $(1324)(56)$

7. (6 points) Multiply the permutations (write as a product of disjoint cycles):

- $(1324) \cdot (243)$

- $(13256)(478) \cdot (14)(278)(56)$

8. (4 points) Give an example of a finite nonabelian group.

9. (4 points) Give an example of an abelian subgroup of a nonabelian group.

10. (6 points) Give a reason why each pair of groups cannot be isomorphic?

- $\mathbb{Z}_9$ and $U(9)$

- S_4 and $\mathbb{Z}_{24}$

11. (6 points) Identify each permutation as even or odd.

- (1234)

- (12)(34)

- (12)(13)(14)

12. (5 points) Draw the subgroup lattice of the group $\mathbb{Z}_{20}$.

13. (5 points) How many automorphisms are there of the group $\mathbb{Z}_9$?

14. (6 points) How many subgroups does $\mathbb{Z}_{27}$ have? (a) List a generator and (b) give the order for each of these subgroups.

Proofs. Give clear arguments. Don't skip steps.

15. (8 points) Suppose that a group G has two elements of order three which are *not* inverses of one another. Prove that G is *not* cyclic.

16. (8 points) Prove that the map $f : S_n \to S_n : f(\sigma) = \sigma \cdot \sigma$ cannot be an isomorphism for any $n > 1$.

17. (8 points) Let H be the cyclic subgroup of $\mathbb{Z}$ generated by 7. Prove that the map $f : \mathbb{Z} \to H : f(z) = 7z$ is an isomorphism.