Math 3350. Test 3 (Harvey Spring 2012).

Name (5 points): ______________________________

[No books or notes are allowed. Please show all of your work.]

1. (6 points) State Lagrange's Theorem.

2. (6 points) Definition: A map $f : G \to H$ is an isomorphism if:

3. (6 points) What are the elements of the group $Aut(G)$ (describe in words)? What is the operation?

4. (6 points) A group has order 30. What are the possible orders of its subgroups?

5. (9 points) What are the orders of the following groups?

(a) $A_4 \oplus \mathbb{Z}_{12}$

(b) $\mathbb{Z}_4 \oplus \mathbb{Z}_4$

(c) $U(8) \oplus \mathbb{Z}_8$

6. (6 points) Give an example of two finite non-abelian groups that are not isomorphic.

7. (8 points) What are the orders of the elements?

   (a) $(4, 3)$ in $\mathbb{Z}_6 \oplus \mathbb{Z}_6$

   (b) $((123), 5)$ in $S_4 \oplus \mathbb{Z}_{20}$

8. (6 points) List all of the cosets of $\langle 17 \rangle$ in the group $U(18)$. Hint: $17 \equiv -1 \mod 18$.

9. (6 points) Give an example of two groups with order 16 that are not isomorphic.

10. (6 points) Are the following groups cyclic? Explain briefly why or why not.

(a) $\mathbb{Z}_3 \oplus \mathbb{Z}_5$?

(b) $\mathbb{Z}_3 \oplus \mathbb{Z}_6$?

11. (10 points) Let G be a group with the property that $a^3 = a$ for all $a \in G$, and suppose that $f : G \to H$ is an isomorphism. Prove that $a^3 = a$ for all $a \in H$ as well.

12. (10 points) Prove that $\mathbb{R} \oplus \mathbb{R} \simeq \mathbb{C}$. [Note that $\mathbb{C}$ denotes the set of complex numbers of the form $a + bi$, with $i = \sqrt{-1}$. It forms a group with the complex addition operation $(a+bi)+(c+di) = (a+c)+(b+d)i$.]

13. (10 points) Let G be a non-cyclic group of order pq, where p and q are primes. Suppose that $a^q \neq e$. Prove that $a^p = e$.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

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