MATH 3350. TEST 3 (HARVEY SPRING 2013).

Name (2 points): ____

[No books, notes, or calculators are allowed. Please show all of your work.]

Definitions. Be precise and concise. Do not give vague answers.

1. (6 points) State Lagrange's Theorem.

2. (6 points) Let H be a subgroup of G, and let $a \in G$. The left coset aH is:

3. (6 points) A subgroup H of G is a *normal* subgroup if:

4. (6 points) For groups G and H, the direct product $G \oplus H$ consists of the set:

   together with the operation:

5. (6 points) Let H be a normal subgroup of G. The factor group G/H consists of the set:

together with the operation:

Problems. Short answers are sufficient.

6. (8 points) Is $\mathbb{Z}_2 \oplus \mathbb{Z}_6 \simeq \mathbb{Z}_{12}$? Why or why not?

7. (8 points) List the left cosets of $H = \langle (1, 1) \rangle$ in $\mathbb{Z}_3 \oplus \mathbb{Z}_3$.

8. (8 points) What is the order of the element $((12)(34), 3)$ in $A_4 \oplus \mathbb{Z}_{12}$?

9. (8 points) What is $[S_4 : \langle (1234) \rangle]$, the index of the subgroup generated by (1234) in S_4?

10. (8 points) Give an example of a group G and a subgroup H which is *not* a normal subgroup of G.

Proofs. Give clear arguments. Don't skip steps.

11. (10 points) Prove that every non-cyclic group of order four is abelian (note that a cyclic group is always abelian, so this means that *all* groups of order four are abelian).

12. (10 points) Let G be a group, and let K be the subgroup of $G \oplus G \oplus G$ which consists of all elements with the same first two coordinates. That is,

$$K = \left\{ (g_1, g_2, g_3) \in G \oplus G \oplus G \;\middle|\; g_1 = g_2 \right\}.$$

Prove that $K \simeq G \oplus G$.

13. (10 points) Suppose that K and H are both normal subgroups of G. Then H/K is a subgroup of G/K (this is easy, but you don't have to prove it). Prove that H/K is a *normal* subgroup of G/K.