Math 3400. Final Exam (Harvey Summer 2010).

Name (5 points):

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1 (6 points) Compute the gradient of the function $f(x, y, z) = x + yz + \sin(xz)$.

2 (5 points) The Cartesian coordinates of a point are $(1, -1, 2)$. What are the cylindrical coordinates of this point?

3 (6 points) Compute the Jacobian of the transformation

$$\begin{cases} x = u^2 - v^2 \\ y = uv \end{cases}$$

4 (10 points) Let $\mathbf{r}(t) = \langle \cos(3t), \sin(3t), -4t \rangle$.

(a) Compute the unit tangent vector $T(t)$.

(b) Compute the unit normal vector $N(t)$.

5 (10 points) Sketch each of the solids.

(a) $\{0 \leq \theta \leq 2\pi,\ 1 \leq r \leq 2,\ 0 \leq z \leq 1\}$

(b) $\{0 \leq \rho \leq 1,\ 0 \leq \theta \leq \pi/2,\ 0 \leq \phi \leq \pi/2\}$

6 (6 points) Sketch the region of integration and change the order of integration in the following integral

$$\int_0^1 \int_{x^3}^{\sqrt{x}} f(x,y)\,dydx.$$

7 (10 points) Write the double integral $\iint_D f(x,y)\,dA$ as an iterated integral for each region D described below.

(a) $D = \{0 \le x \le 4,\ x \le y \le 8 - x\}$.

(b) D is the region inside the circle centered at $(0,0)$ with radius 2.

8 (6 points) Write the triple integral $\iiint_D f(x,y,z)\,dV$ as an iterated integral where D is the region inside the cylinder $x^2+y^2=1$ and between the planes $z=0$ and $z=x+y+2$.

9 (10 points) Set up the integral and simplify the integrand. You do not have to evaluate the integral.

(a) $\int_C x^2\,dx + xy\,dy$, where the curve C is parametrized by $\mathbf{r}(t)=\langle t,t^3\rangle$, $0\le t\le 2$.

(b) $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}=\langle x^2,xy\rangle$ and C is parametrized by $r(t)=\langle \ln t, t\rangle$, $2\le t\le 5$.

10 (10 points) Set up the double integral and simplify the integrand. You do not have to evaluate the integral.

(a) $\iint_S xz\, ds$, where S is the portion of the plane $x + y - z = 1$ where $0 \leq x \leq 2$ and $0 \leq y \leq 3$.

(b) $\iint_S \mathbf{F} \cdot d\mathbf{s}$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and S is the portion of the paraboloid $z = x^2 + y^2$ between $z = 0$ and $z = 1$.

11 (10 points) Let $\mathbf{F} = x^2y\mathbf{i} + yz\mathbf{j} + x\mathbf{k}$. Compute $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.

12 (8 points) Let $\mathbf{F}(x) = -x\mathbf{i} + 2\mathbf{j} + xz\mathbf{k}$ and let C be the edge of the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 2$, $z = 1$ traversed in the counterclockwise direction. Use Stokes's Theorem to set up a double integral which will calculate

$$\oint_C \mathbf{F} \cdot d\mathbf{r}.$$

You do not have to evaluate the integral.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

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