Math 3400. Test 1 (Harvey Summer 2010).

Name:

---

1 (10 points) (a) Find the cylindrical coordinates of the rectangular point $(-\sqrt{2}, -\sqrt{2}, 5)$.

(b) Find the spherical coordinates of the rectangular point $(1, 1, 0)$.

2 (10 points) (a) Translate the equation from rectangular to cylindrical coordinates:

$$z = 4x^2 + 4y^2.$$

(b) Translate the equation from spherical to rectangular coordinates:

$$\rho = 3 \sec \phi.$$

3 (10 points) Sketch the solid whose cylindrical coordinates satisfy the inequalities

$$0 \leq r \leq 2 \quad 3\pi/2 \leq \theta \leq 2\pi \quad 0 \leq z \leq 4.$$

4 (10 points) Find the equation of the tangent line to the curve

$$\mathbf{x}(t) = \langle \cos t, \sin(2t), t \rangle$$

when $t = \pi/4$.

5 (10 points) Find the arc length of the curve

$$\mathbf{x}(t) = \langle \cos(3t), \sin(3t), 4t \rangle$$

between $t = 0$ and $t = 2\pi$.

6 (10 points) In this problem, you will need to use the formula for a parabolic trajectory

$$\mathbf{x}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v_0} + \mathbf{x_0}$$

where g is the acceleration due to gravity, $\mathbf{v_0}$ is the initial velocity and $\mathbf{x_0}$ is the initial position. If a projectile is fired from level ground at an angle of $45°$ with an initial speed of $100\, ft/sec$, how far away will the projectile land?

7 (10 points) Compute the gradient of the function $f(x, y, z) = x^3 - 4xy^2 + y + e^z$.

8 (10 points) Let $\mathbf{F} = (x + y)\mathbf{i} + (y + z)\mathbf{j} + (x + z)\mathbf{k}$.

(a) Compute $\nabla \cdot F$

(a) Compute $\nabla \times F$

9 (20 points) Consider the curve:

$$\mathbf{r}(t) = \langle 4t, \sin(3t), -\cos(3t) \rangle.$$

(a) Compute $T(t)$, the unit tangent vector.

(b) Compute $N(t)$, the principal normal vector.

(c) Find the curvature of $r(t)$ when $t = 1$.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

$\times$ ______________________________