MATH 3400. TEST 2 (HARVEY SUMMER 2010).

Name (5 points):

1 (12 points) Calculate each iterated integral.

(a)
$$\int_0^1 \int_0^2 (x^2 + y)\, dy\, dx$$

(b)
$$\int_0^1 \int_x^1 (x - y)\, dy\, dx$$

2 (12 points) Write each double integral as an iterated integral. You do not need to evaluate the integrals.

(a)

$$\iint\limits_D f(x,y)\,dA \quad \text{where } D = \left\{(x,y)\,\middle|\,0 \le x \le 1,\ 2 \le y \le 5\right\}.$$

(b)

$$\iint\limits_D f(x,y)\,dA \quad \text{where } D = \left\{(x,y)\,\middle|\,1 \le x \le 3,\ 1+x \le y \le 8-x\right\}.$$

3 (12 points) Sketch the region of integration and change the order of integration. You do not need to evaluate the the integral.

$$\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx.$$

4 (10 points) Compute the Jacobian of the transformation $(x, y, z) = T(r, \theta, w)$ where

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = \ln(w^2 + 1). \end{cases}$$

5 (12 points) Use a change of variables to polar coordinates to evaluate the integral

$$\int_{-\sqrt{\pi}}^{\sqrt{\pi}} \int_{-\sqrt{\pi - x^2}}^{\sqrt{\pi - x^2}} \sin(x^2 + y^2) \, dy \, dx.$$

6 (18 points) Write the triple integral $\iiint\limits_{W} f(x, y, z) \, dV$ as an iterated integral for each of the regions W specified below.

(a) $W = \left\{ (x, y, z) \middle| 0 \le x \le 3, \ 0 \le y \le 6 - 2x, \ 1 \le z \le x + y + 2 \right\}.$

(b) W is the region in the first octant $(x, y, z \ge 0)$ below the plane $x + y + z = 4$.

(c) W is the region inside the cylinder $x^2 + y^2 = 4$, above the plane $z = 0$, and below the paraboloid $z = 10 - x^2 - y^2$.

7 (10 points) Write a *double* integral to calculate the area inside the circle $x^2 + y^2 = 9$. You may use either rectangular or polar coordinates.

8 (12 points) Consider the double integral $\iint\limits_{D} xy\, dA$ where D is the region of the plane which is above the parabola $y = x^2$ and below the line $y = 4$.

(a) Set up the iterated integral with $dA = dx\, dy$.

(b) Set up the iterated integral with $dA = dy\, dx$.