MATH 3400. TEST 3 (HARVEY SUMMER 2010).

Name (5 points):

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1 (32 points) Set up an appropriate integral to evaluate the line or surface integral. Simplify the integrand but do not evaluate you integral.

(a) The scalar line integral $\int_C (x+y)\,ds$ over the parabolic path C parametrized by $\mathbf{r}(t) = \langle t, t^2 \rangle$, $0 \le t \le 1$.

(b) The integral $\int_C \mathbf{F} \cdot d\mathbf{s}$ where $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} - xy\mathbf{k}$ and C is the circular path parametrized by $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), 4 \rangle$, $0 \le t \le \pi$.

(c) The surface integral $\iint\limits_S xy\,dS$ over the surface S parametrized by $\langle s, t, s{+}t{+}1\rangle$, $0 \le s \le 1$, $0 \le t \le 2$.

(d) The surface integral $\iint\limits_S z\,dS$ over the surface S which consists of the portion of the cylinder $x^2 + y^2 = 1$ between $z = 3$ and $z = 6$.

2 (10 points) Let C be the circle $x^2 + y^2 = 9$ traversed counterclockwise and consider the vector line integral

$$\int_C -x^2 y\, dx + y^2\, dy$$

Use Green's Theorem to set up an equivalent double integral. You do not have to evaluate it.

3 (15 points) Give a parametrization for each surface. Be sure to include the bounds for your parametrization.

(a) The surface $z = 2x^2 + 3y$, $0 \le x \le 1$, $0 \le y \le 1$.

(b) The top half of the sphere of radius 1 ($x^2 + y^2 + z^2 = 1,\ z \geq 0$).

(c) The unit disk in the xy-plane: $x^2 + y^2 \leq 1,\ z = 0$.

4 (15 points) (a) Give the formula for Green's Theorem.

(b) Give the formula for Stokes's Theorem.

(c) Give the formula for Gauss's Theorem.

5 (12 points) Let $\mathbf{F}(x) = x\mathbf{i} + y\mathbf{j} + xy\mathbf{k}$ and let S be the sphere $x^2 + y^2 + z^2 = 1$. Use Gauss's Theorem to set up a *triple* integral which will calculate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

You do not have to evaluate the integral.

6 (12 points) Let $\mathbf{F}(x) = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ and let C be the boundary of the disk $x^2 + y^2 = 1$, $z = 3$ traversed in the counterclockwise direction. Use Stokes's Theorem to set up a double integral which will calculate

$$\oint_C \mathbf{F} \cdot d\mathbf{S}.$$

You do not have to evaluate the integral.

I certify as a student at The University of Virginia's College at Wise that I have neither received nor given aid on this test.

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