Twenty-Third Souteastern-Atlantic Regional Conference on Differential Equations

Abstracts of Papers Presented

October 2003

List of SEARCDE Abstracts, 2003

Phototransduction Modeling

Vasilios.Alexiades
Univ. of Tennessee and ORNL
H.Khanal
Emry-Riddle Aeronautical Univ.

Abstract

Phototransduction is the process by which light is converted into an electrical response in retinal rod and cone photoreceptors. We present a hierarchy of models for interaction and diffusion of cGMP (cyclic Guanosine Monophosphate) and Ca2+ (calcium ions) in the cytoplasm of vertebrate rod photoreceptors. The models include a bulk (ODEs) model and three spatially resolved (PDEs) models: 1-D longitudinal, 3-D full model, and a homogenized limit of the full model. Numerical simulations of the response to a single photon from each model will be presented and compared.

Bifurcating Rotating Wave Solutions of the FitzHugh-Nagumo Equations on a Ring

John Alford
Mathematics Department
Tulane University
6823 St. Charles Ave.
New Orleans, La 70118

Abstract

The FitzHugh-Nagumo equations are a nonlinear system of reaction-diffusion equations in two variables. They are commonly used as a model for the behavior of nerve cells that are subjected to a stimulus. In particular, they have been used to model cardiac arrhythmias that take the form of a self-propagating electrical pulse that rotates around a ring of heart tissue.

This talk will show some results from computations of bifurcating rotating wave solutions of the FitzHugh-Nagumo equations (with uniformly applied current stimulus term) on a ring. I will also discuss how bifurcation theory can be

used to describe some characteristics of the onset of these waves with regards to dependence on stimulus intensity, ring radius, and size of the diffusion coefficient.

Mathematical modeling and analysis for system of Maxwell's equations

Kumud Altmayer

University of Arkansas at Pine Bluff, Mathematical Sciences

and Technology Dept., Pine Bluff, AR 71601 USA

Email: altmayer_k@uapb.edu

AMS (MOS) subject classification. 65N12, 65N30

Abstract

This talk/presentation deals with the study of Maxwell's equations for homogenous, or nonhomogenous medium with Dirichlet and Mixed Boundary conditions. We will study time independent equations first and give ideas for generalizations to that of time dependent equations. Many cases of time dependent as well as time independent Maxwell's equations have been studied before. We analyze further for stable solutions. Then a comparison of exact solutions to numerical solutions is established. An example is under consideration that will depict how equations and solutions behave differently in different media/materials. A combination of Finite volume method and Mortar element method are being used as tool for obtaining the solutions, error estimates in L^2 and L^p estimates.

Vortex Sheets with Surface Tension

David Ambrose

Courant Institute

251 Mercer St., New York, NY 10012

ambrose@cims.nyu.edu

Abstract

A vortex sheet is the interface between two incompressible, inviscid, irrotational fluids moving past each other. The classical evolution equations modeling the motion of a vortex sheet do not account for surface tension, and they are ill-posed (both linearly and nonlinearly). The linearization of the equations of motion becomes well-posed, however, when surface tension is accounted for. Thus, it has been conjectured that the full vortex sheet problem with surface tension is well-posed. In this talk, I will discuss my proof of this conjecture. The proof uses ideas from the numerical work of Hou, Lowengrub, and Shelley.

Some New Results on Controllability of Mechanical Models

Sergei A. Avdonin, University of Alaska Fairbanks, AK, USA,

Boris P. Belinskiy (speaker), University of Tennessee at Chattanooga, TN, USA

Abstract

The classical papers by H.O. Fattorini and D.L. Russell have established a close connection between the problem of exact controllability of the (linear) systems with distributed parameters on the time interval [0,T] and the classical problem of moments for non-harmonic exponentials. We present two developments of the afore-mentioned results for the simplest model of an elastic string. (1) If the tension of the string is a function of time, then the non-harmonic exponentials have to be substituted by some functions which even may not be found explicitly. We prove that, under some assumptions, they form a basis in the corresponding Hilbert space and hence solve the moment problem and the problem of exact controllability. (2) If the string is rotating, the corresponding family of non-harmonic exponentials does not form a Riesz basis for any time T. However, using recent results about exponential bases in Sobolev spaces with non-integer indices, we show that the string is controllable in the appropriate sense.

Global Existence and Regularity for the Lagrangian Averaged Navier-Stokes equations with Initial Data in $H^{1/2}$

Joel Avrin

Department of Mathematics

University of North Carolina at Charlotte

Charlotte, North Carolina 28223

jdavrin@email.uncc.edu

Abstract

We consider the Lagrangian averaged Navier-Stokes (LANS-α) equations on a bounded domain in R^3 with zero (no-slip) boundary conditions. With periodic boundary conditions on a box, these equations are also known as the Camassa-Holm equations. The (LANS-α) model averages or coarse-grains the small, computationally unreasonable, scales of the Navier-Stokes equations; spatial scales smaller than $\alpha > 0$ are averaged out. We establish the existence and uniqueness of local strong (i.e., regular) solutions with initial data in $H^{1/2}$, and then use the a priori estimate developed in [1] to conclude that these are global regular solutions. Our results extend those in [2] and [1], which show the global well-posedness of H^1 weak solutions in a periodic box and on a bounded domain with no-slip boundary conditions, respectively.

[1] D. Coutand, J. Peirce, and S. Shkoller, Global Well-Posedness of Weak Solutions for the Lagrangian-Averaged Navier-Stokes Equations on Bounded Domains, Comm. Pure Appl. Anal. **1** (2002), 35-50.

[2] C. Foias, D.D. Holm and E.S. Titi, The Three Dimensional Viscous Camassa-Holm Equations and Their Relation to the Navier-Stokes Equations and Turbulence Theory, J. Dyn. Diff. Eq., (1999), to appear.

A Study of LES Models Using Sensitivity Analysis

Jeff Borggaard,

Interdisciplinary Center for Applied Mathematics

Department of Mathematics,

Virginia Tech

Abstract

As a test bed for studying large eddy simulation (LES) modeling techniques, we begin with Burgers equation and perform the usual filtering/sub grid scale operations. Burgers equation has long been used as a simplified version of Navier Stokes equations and has a "high Reynolds number" analogue. Not only does the simplicity of Burgers equation lead to rapid simulations, but in many instances, we can compute a closed form solution. This allows us to isolate multi scale modeling issues such as boundary conditions, subgrid scale models and filters. Our approach is to use sensitivity analysis to compare the sensitivity of solutions obtained with various closure models to model parameters. Examples of parameters are filter width, coefficients in eddy viscosity models, and boundary conditions in deconvolution models. The latter parameters being associated with closure models. Insensitivity of the solution to these parameters is considered to be a desirable trait, since these values are obtained using mathematical processes rather than fitting experimental data.

The fingering problem in a Hele-Shaw cell

Andreucci, D., Caruso, G, DiBenedetto, E.

University of Rome Tor Vergata

Abstract

The Hele-Shaw fingering problem is a classical problem arising when two immiscible viscous fluids move inside a Hele-Shaw cell, consisting of two horizontal, slightly separated, ideally infinite parallel plates sealed at their edges. The cell is initially filled with a viscous fluid, which is than displaced by a less viscous fluid forced into the cell. In the experiments, after a brief transient period,

the penetrating fluid reaches a steady-state configuration taking the form of a long finger, symmetric with respect to the axis of the cell and advancing with a constant speed. Moreover the asymptotic thickness of this steady finger is observed to be equal to a half of the cell width. The explicit solutions of this problem found by Saffman and Taylor exhibit a lack of uniqueness, as a solution is possible for each assigned asymptotic thickness of the finger. Here the problem is casted as an obstacle-like problem, trough a suitable variational formulation. The existence of weak solutions is proved, allowing fingers of a quite irregular shape. It turns out that a unique solution exists for each prescribed asymptotic shape of the finger, up to a translation along the cell axis. Moreover the weak formulation gives new insights for a physical selection criterium for the classical Saffman-Taylor solutions.

Boundary Determination in Inverse Initial-Boundary Value Problems

Lester F. Caudill, Jr.
University of Richmond
Richmond, VA 23173

Abstract

We discuss inverse problems of boundary determination for parabolic initial-boundary value problems. In this setting, the desired unknown is a portion of the boundary of the spatial domain. This unknown is to be determined from a single Cauchy data pair prescribed on another portion of the boundary. This type of inverse problem models the use of thermal methods in nondestructive damage assessment. This specific problem could represent a model of thermal imaging, in which an inaccessible portion of the boundary of a sample is to be estimated by temperature measurements (resulting from an induced heat flux pattern) taken on another portion of the boundary.

We discuss uniqueness for this inverse problem, and important stability issues. We present a numerical algorithm designed to produce stable and reliable approximate solutions to this problem. We also discuss a number of important and interesting issues, both of a theoretical and practical nature, that arise in the study of this inverse problem.

Existence Results for Singular Three Point Boundary Value Problems on Time Scales

Jeffrey J. DaCunha
Dept. of Mathematics
Baylor University
Waco, Texas 76798

Abstract

The existence of a positive solution is obtained for the second order threepoint boundary value problem,

$$y^{\Delta\Delta} + f(x, y) = 0,$$

$x \in (0, 1] \cap \mathbb{T}$, $y(0) = 0$, $y(p) = y\left(\sigma^2(1)\right)$ where $p \in (0, 1) \cap \mathbb{T}$ is fixed and where $f(x, y)$ is singular at $y = 0$, and possibly at $x = 0$, $y = \infty$. The method applies a fixed point theorem (due to Gatika, Oliker, and Waltman) for mappings that are decreasing with respect to a cone.

Introducing a MATLAB Numerical Toolbox for Dynamic Equations on Time Scales

John Davis
Baylor University
Waco, Texas

Abstract

In this talk, we will present the first numerical toolbox for analyzing dynamic equations on time scales. We will showcase how this toolbox can be used to plot various time scales (whether canonical and user defined), basic functions on time scales (e.g. $\sigma(t)$, $\rho(t)$, $\mu(t)$), as well as more advanced functions such as $e_p(t, t_0)$, $\cosh_p(t, t_0)$, $\sinh_p(t, t_0)$, $\cos_p(t, t_0)$, $\sin_p(t, t_0)$, and others. We will also present the numerical and graphical capabilities of our dynamic equation solver for first order systems.

Boundedness and Stability in Nonlinear Discrete Systems with Nonlinear Perturbation

Youssef M. Dib and Youssef N. Raffoul
Department of Mathematics,
University of Dayton,

Dayton OH 45469-2316

Abstract

We consider the nonlinear Volterra discrete system with nonlinear perturbation

$$x(n+1) = A(n)x(n) + \sum_{s=0}^{n} B(n,s) f(s, x(s)) + g(n, x(n)).$$

Our goal is to use Lyapunov functionals to obtain conditions that guarantee all solutions of the above Volterra equation are bounded, and derive conditions that ensure asymptotic stability and exponential stability, in the case $x = 0$ is a solution.

On Nonoscillatory Half-linear Differential Equations

Zuzana Došlá

Masaryk University Brno

Czech Republic

Abstract

We present some recent results achieved in the joint research with M. Cecchi and M. Marini of University of Florence for the half-linear differential equation

$$(a(t)\Phi_p(x'))' = b(t)\Phi_p(x) \tag{0.1}$$

where functions a, b are continuous for $t \geq 0$, $a(t) > 0$ and $\Phi_p(u) = |u|^{p-2}u$ with $p > 1$.

The notion of principal solution of the nonoscillatory linear equation was introduced by W. Leighton, M. Morse and P. Hartman as a "smallest solution in a neighbourhood of infinity". Following the Riccati equation approach, the notion of principal solution has been extended to (0.1) independently by J. Mirzov and by A. Elbert–T. Kusano. We show that the limit and integral characterizations of principal solutions hold for (0.1).

Kelvin Helmholtz Instability Waves in Supersonic Multiple Rectangular Jets with Large Ratio

Joshua Z.Du

Department of Mathematics

Kennesaw State University

Kennesaw, GA 30144 5591

Abstract

Three physics laws, Conservation of Mass, Momentum and Energy in differential form, are used to formulate the Kelvin Helmholtz Instability wave problem of supersonic rectangular multiple jets with large ratio of the height versus width of each rectangular jet. A dispersion relation for studying the Kelvin Helmholtz Instability and Acoustic Wave is derived. Numerical solutions of the eigenvalues and eigenfunctions for the upstream propagating acoustic waves and the Kelvin Helmholtz instability waves can be found by the traditional Newton Iteration Method and the Trace Theorem.
AMS (MOS) Subject Classification 76J20, 76E99

An Existence Result for a First Order Impulsive Functional Differential Inclusion with Variable Times

John R. Graef
Department of Mathematics
University of Tennessee at Chattanooga
Chattanooga, TN 37403-2504 USA
and
M. Benchohra and A. Ouahabi
Department of Mathematics
University of Sidi Bel Abbes
BP 89 2000 Sidi Bel Abbes, Algeria

Abstract

The authors consider the first order impulsive functional differential inclusion with variable times,

$$y'(t) - \lambda y(t) \in F(t, y_t), \ \ a.e.\ t \in J = [0,T],\ t \neq \tau_k(y(t)), \ \ k = 1, \ldots, m,$$

$$y(t^+) = I_k(y(t^-)), \ \ t = \tau_k(y(t)), \ \ k = 1, \ldots, m,$$

$$y(t) = \phi(t), \ \ t \in [-r, 0],\ y(0) = y(T),$$

where $W = 2^{\mathbb{R}^n}$ is the family of all subsets of $\mathbb{R}^n$,

$D = \{\psi\,[-r,0] \to \mathbb{R}^n \,|\, \psi$ is continuous everywhere except for a finite number of points $\bar{t}$ at which $\psi(\bar{t})$ and $\psi(\bar{t}^+)$ exist and $\psi(\bar{t}^-) = \psi(\bar{t})\}$,

$\lambda \in \mathbb{R}^+$, $F : [0,T] \times D \to W$ is a compact convex valued multi-valued map, $I_k \in C(\mathbb{R}^n, \mathbb{R}^n)$, $\tau_k \in C^1(\mathbb{R}^n, \mathbb{R})$, $k = 1, 2, \ldots, m$, $\phi \in D$, and $0 < r < \infty$.

For any y defined on $[-r, T]$ and any $t \in J$, $y_t(\theta) = y(t+\theta), \ \ \theta \in [-r, 0]$. They prove the existence of solutions using Martelli's fixed point theorem for condensing multi-valued maps.

Solving An Infinite Dimensional Stochastic Optimal LQR Control Problem

Cavit Hafizoglu
Department of Mathematics,
University of Virginia, Kerchof
Hall Charlottesville,VA 229003

Abstract

The goal of the talk is to solve the infinite dimensional stochastic linear quadratic control problem under the conditions that the control operator B is bounded and the state operator A generates a semigroup, not necessarily an analytic semigroup, like in the papers Flandoli, DaPrato. The existence and the uniqueness will be proved and the representation of the optimal control and the value function will be presented by solving the Differential Riccati Equation.

Theoretical challenges arising from the questions of controllability for elastic structures

Mary Ann Horn
Vanderbilt University

Abstract

In the context of control of elastic systems, the challenge of obtaining rigorous theoretical results has given rise to the development of new mathematical methods and expanded applications of known tools. This talk will give an overview of the questions of controllability and stabilizability for plates, cylindrical shells and three-dimensional elasticity, as well as the issues arising when single components are coupled into a more complex system. The need for and the utility of high-powered mathematical tools such as microlocal analysis, Carleman estimates and Riemannian geometry will be illustrated and examples of some of the current challenges will be discussed.

Boundedness and Stability In Volterra Equations

Muhammad Islam
Professor & Graduate Program Director

Department of Mathematics
University of Dayton
300 College Park
Dayton, OH 45469 2316

Abstract

Sufficient conditions for the boundedness of the solutions as well as various stability properties of the zero solution of Volterra integrodifferential equations are studied. Liapunov's direct method is used in the analysis as the principal mathematical tool.

Numerical solutions to a semilinear elliptic equation on an annulus

Samuel N. Jator
Austin Peay State University
601 College Street
Clarksville, TN 37044

Abstract

We present numerical positive radial solutions to the semilinear elliptic equation subject to the Dirichlet boundary condition on an annulus $\mathbf{R}^3$.

Solving a Crop Problem by an Optimal Control Method

Hem Raj Joshi
3800 Victory Parkway
Department of Math and CS
Xavier University
Cincinnati, OH 45207

Abstract

A system of ODE coupled with a PDE has been studied in order to understand the spread of pathogen. Two different types of crops are planted in same field in some pattern so that the spread of pathogen can be controlled. The pathogen prefers to eat one crop and the other crop, which is not preferred by pathogen, is introduced to control the spread of pathogen in the farming land.

Comparison Theorems for m-Point Boundary Value Problems

Basant Karna
Department of Mathematics
Baylor University
Waco, Texas 76798-7328, USA
Basant_Karna@baylor.edu

Abstract

We compare smallest eigenvalues of two different types of m-point boundary value problems for some fourth order linear ordinary differential equations. We then extend our results to m-point boundary value problems for even order linear ordinary differential equations.

A Singular Conjugate Boundary Value Problem on a Time Scale

Eric R. Kaufmann and Nickolai Kosmatov
Department of Mathematics and Statistics
University of Arkansas at Little Rock
Little Rock, Arkansas 72204-1099

Abstract

Let S be a time scale symmetric about $1/2$. Let $1/2 \in S$ be right dense and define $T = S \bigcap [0,1]$. The conjugate nonlinear boundary value problem,

$$\begin{aligned} -u^{\Delta\Delta}(t) &= a(t)f(u(t)), t \in \mathbb{T} \setminus \{0,1\}, \\ u(0) &= u(1) = 0, \end{aligned}$$

where $a(t)$ is singular at $t = 1/2$ and f satisfies certain growth conditions, is shown to have infinitely many solutions using Krasnosel'skiĭ's fixed point theorem.

Solution Matching for a Third Order Equation on a Time Scale

Eric R. Kaufmann, Nickolai Kosmatov Martin Eggensperger
Department of Mathematics and Statistics
University of Arkansas at Little Rock
Little Rock, Arkansas 72204-1099

Abstract

Let $\mathbb{T}$ be a time scale such that $t_1, t_2, t_3 \in \mathbb{T}$. We show the existence of solutions for the three-point boundary value problem

$$\begin{aligned} y^{\Delta\Delta\Delta}(t) &= f(t, y, y^{\Delta}, y^{\Delta\Delta}),\ t \in \mathbb{T} \\ y(t_1) &= y_1,\ y(t_2) = y_2,\ y(t_3) = y_3, \end{aligned}$$

by matching solutions to the two two-point boundary value problems

$$y^{\Delta\Delta\Delta}(t) = f(t, y, y^{\Delta}, y^{\Delta\Delta}),$$

$t \in \mathbb{T},\ y(t_1) = y_1,\ y(t_2) = y_2$, and

$$y^{\Delta\Delta\Delta}(t) = f(t, y, y^{\Delta}, y^{\Delta\Delta}),$$

$t \in \mathbb{T},\ y(t_2) = y_2,\ y(t_3) = y_3$.

Alternative Calculus Generated by a Differential Equation

Zviad Khukhunashvili
Tougaloo College

Abstract

We investigate an algebraic structure of the space of solutions of autonomous nonlinear differential equations of certain type. It is shown that there are infinitely many binary algebraic laws of addition of solutions. Among these we extract commutative and conjugate commutative groups that lead to the conjugate differential equations. Group homomorphic relations are established between quasilinear and linear systems. A commutative algebraic object that is a pair of two alternative fields with common identity elements is derived. We study those mathematical constructions that are related to the existence of these fields, in particular, differential and integral calculus based on the commutative algebra that is generated by a given differential equation. It turns out that along with the standard calculus there always exist isomorphic alternative calculi. Moreover, every system of differential equations generates its own calculus that is isomorphic (or homomorphic) to the standard one. The given system written in its own

calculus appears written in its own calculus appears to be linear. The notion of inner and outer times is introduced.

Numerical Methods for Viscous and Nonviscous Wave Equations

Hyeona Lim, Seongjai Kim, and Jim Douglas Jr.

Mississippi State University

Abstract

In this talk, we discuss accurate and efficient numerical methods for solving viscous and nonviscous wave equations. A three-level second-order implicit algorithm is considered without introducing auxiliary variables. As a perturbation of the algorithm, a locally one-dimensional (*LOD*) procedure which has a splitting error not larger than the truncation error is suggested to solve problems of diagonal diffusion tensors in cubic domains efficiently. Both the three-level algorithm and its *LOD* procedure are proved to be unconditionally stable. An error analysis is provided for the numerical solution. Numerical results are presented to show the accuracy and efficiency of the new algorithms for the propagation of acoustic waves and of microscale heat transfer.

Method of Undetermined Coefficients for the Analytic Solutions of Iterative Functional Differential Equations

Yin-Wei Lin and Tzon-Tzer Lu

Department of Applied Mathematics

National Sun Yat-sen University

Kaohsiung 80424, Taiwan, R.O.C.

Abstract

Functional differential equations with delay have long been studied due to their extensive applications. Among them many researches study the case when the deviating argument depending on the state variable. For example, E. Eder in 1984 discussed the iterative functional differential equation $x'(t) = x(x(t))$. Recently great attention has been paid to the analytic solutions of such iterative functional differential equations, e.g. the works by S.S. Cheng, J.G. Si and X.P. Wang, etc. However, their method seems complicated and not straight forward. For $x'(t) = x(x(t))$, in fact, they seek the power series solution of the form $x(z) = y(\alpha y^{-1}(z))$ by solving the auxiliary equation $y'(\alpha z) = \frac{1}{\alpha} y'(z) y(\alpha^2 z)$. In this talk we use the well known method of undetermined coefficients to obtain the analytic solutions of such iterative functional differential equations of first

order and second orders. Let's denote $x^{[m]}(z) = x(x(\cdots x(z)))$ the m-th iterate of the function $x(z)$. The classes of differential equations we solve include

$$x'(z) = x^{[m]}(z),$$

$$x'(z) = x(az + bx(z)),$$

$$\alpha z + \beta x'(z) = x(az + bx'(z)),$$

$$x'(x^{[r]}(z)) = c_0 z + c_1 x(z) + \cdots + c_m x^{[m]}(z),$$

$$x'(z) = \frac{1}{c_0 z + c_1 x^{[1]}(z) + \cdots + c_m x^{[m]}(z)},$$

$$x''(z) = (x^{[m]}(z))^2,$$

$$x''(z) = x(az + bx(z)),$$

$$x''(x^{[r]}(z)) = c_0 z + c_1 x(z) + \cdots + c_m x^{[m]}(z),$$

etc.

Key Words: iterative functional differential equation, analytic solution, Taylor series method, method of undetermined coefficients, fixed point iteration.

Some New Advances in the Theory of Dynamic Materials

Konstantin A. Lurie (Invited Speaker)

Department of Mathematical Sciences

Worcester Polytechnic Institute

100 Institute Road

Worcester, Massachusetts 01609, U.S.A.

klurie@wpi.edu

Abstract

The talk is focused on special material formations termed the dynamic materials (DM). DM are defined as structures assembled from conventional materials distributed in space-time. If such assemblages occur on a microscale, they become spatio-temporal, or dynamic, composites (DC). When a low frequency disturbance propagates through DC, it perceives this one as a medium with some effective properties detected through homogenization. A discussion

of such properties along with some special effects they produce in material design is the central object of the talk. Such effects include material screening, elimination of a cutoff frequency in waveguides, amplification and generation of waves, compression of impulses, frequency multiplication, and so on. A DM is a linear system with coefficients (material parameters) variable in space and time, and their variability may produce said effects, some of them being typical for the non-linear systems. This is accompanied by an exchange of energy and momentum between DM and the environment.

The energy/momentum transformation in DC is examined in the context of electrodynamics of moving dielectrics. Waves of negative energy may particularly emerge through the material mixing in space-time, and such waves may, in special circumstances, demonstrate instabilities and open the way to power generation. The effective properties of DC are needed for the purpose of optimal layout in dynamics. The bounds for such properties related to the mixtures of two or more original dielectrics will be discussed in the talk for one-dimensional wave propagation without shocks. Such bounds appear to be sharp, i.e. attainable by laminates of multiple rank. Other microstructures may demonstrate quite a different performance, not necessarily characterized by the effective properties. For example, a spatio-temporal checkerboard may create "synchronized waves" in one spatial dimension, i.e. the waves with profiles initially occupying some space in one dimension and eventually being compressed into much smaller intervals that shrink into a point as $t \to \infty$.

These and other features typical for DM and DC will become a focus of this presentation.

Approximate Analytic Solutions to a van der Pol Type Oscillator Equation

Dr. Ronald E. Mickens

Clark Atlanta University

Department of Physics

Atlanta, GA 30314, USA

Research supported in part by grants from DOE and the MBRS-SCORE Program at Clark Atlanta University.

Abstract

We calculate an analytical approximation to the solution of a van der Pol type oscillator equation:

$$\ddot{x} + x^{5/3} = \epsilon(1 - x^{2/3})\dot{x}.$$

The general properties of this second-order, nonlinear ODE will be discussed by Rucker [1]. We use an equivalent linearization technique, combined with the method of first-order averaging to obtain the approximate solution [2]. This procedure allows the determination of the angular frequency and the amplitude for the limit-cycle, as well as the transient behavior for the approach of the solution to the periodic solution. A comparison will also be made with numerical solutions gotten from the application of a nonstandard finite difference method.

References

[1] S. A. Rucker, “Preliminary Study of a van der Pol Type Oscillator Equation," SEARCDE-2003.

[2] R. E. Mickens, “A Combined Equivalent Linearization and Averaging Perturbation Method for Nonlinear Oscillator Equations," Journal of Sound and Vibration (accepted for publication).

Singularities of differential equations of lines of curvature for surfaces in 3-Euclidean space

Navarro Soza Jose Matías

Facultad de Matematicas, UADY

Periferico Norte Tablaje 13615

Apartado Postal 172, Cordemex

Merida, Yucatan, Mexico, 97110

Abstract

In this talk we study the behavior of solutions to differential equations of lines of curvature for surfaces in 3-Euclidean space in a neighborhood of an isolated singular point. These singularities are the umbilical points of the surface. The dynamics of these singularities depending on the local parameters of the surface are described. We determine also the type of singularities that the Gaussian and mean curvature functions have at the isolated umbilical points of these class of surfaces.

Some almost automorphic perturbations of certain abstract differential equations

Gaston N'Guerekata
Morgan State University
Department of Mathematics
Baltimore, MD 21251, USA

Abstract

We are concerned with the differential equation

$$x'(t) + (A + B)\, x(t) = f(t) \qquad (*)$$

where A is an infinitesimal generator of a family of strongly continuous group of operators $T(t)$and B an unbounded linear operator acting in a Hilbert space H. The input f is an almost automorphic function. We use the theory of invariant subspaces to prove that under some appropriate conditions on A and B, every solution to (*) is almost automorphic.

Sub-Riemannian Geometry and Analysis of the Heisenberg Group

Yilong Ni
Yale University

Abstract

The short talk I am going to give is a summary of my thesis work. The Heisenberg group H_1 is the simplest non-commutative nilpotent Lie group. On this group, there is a natural analogue of the Laplacian, Δ_H, which is not elliptic. The geometry associated to this sub-elliptic operator known as sub-Riemannian geometry, is quite different from Riemannian geometry. The purpose of my thesis is to study the sub-Riemannian geometry on the Heisenberg group H_1 by considering H_1 as the limit of a family of Riemannian manifolds.

In my thesis, H_1 is viewed as the boundary of the Siegel domain. A class of hypersurfaces in the Siegel domain can be regarded as copies of H_1. The Riemannian metric on each hypersurface degenerates to the sub-Riemannian metric as the hypersurface approaches the boundary. I studied geodesics in the Siegel domain and on each hypersurface and their relations with geodesics on H_1. I also calculated the heat kernel and Green's function in the Siegel domain and on each hypersurface.

On Some Elliptic Equations of Monge-Amp Arising in Differential Geometry

Vladimir Oliker (Invited Speaker)

Emory University,

Atlanta, Georgia, USA

oliker@mathcs.emory.edu

Abstract

Let $\mathcal{R}^{n+1}(K), n \geq 2,\ K = -1, 0, 1$, be a Riemannian manifold of sectional curvature K and m an integer, $1 \leq m \leq n$. We are interested in establishing conditions for existence of a smooth hypersurface M in $\mathcal{R}^{n+1}(K)$ which is star-shaped relative to some point O $\in \mathcal{R}^{n+1}(K)$ and whose (normalized) elementary symmetric function H_m of principal curvatures of M satisfies the equation

$$H_m = \psi_{|_M}, \tag{0.2}$$

where ψ is a given function in $\mathcal{R}^{n+1}(K)$.

In analytic formulation this problem reduces to a nonlinear, second order elliptic equation of Monge-Ampère type on a unit sphere S^n. In Euclidean space $R^{n+1}(= \mathcal{R}^{n+1}(0))$, earlier results by I. Bakelman and B. Kantor and by A. Treibergs and W. Wei give such conditions when $m = 1$ (the mean curvature case), by V. Oliker when $m = n$ (the Gauss curvature case), and by L. Caffarelli, L. Nirenberg and J. Spruck when $1 < m < n$. For $K \neq 0$ and $m = n$ the problem was investigated by V. Oliker.

In this talk we will discuss the same problem in spaces $\mathcal{R}^{n+1}(K)$ with $K \neq 0$. We will present results showing that if conditions on the behavior of the function ψ are tied together with the behavior of the metric of $\mathcal{R}^{n+1}(K)$ then it is possible to obtain some a priori estimates for solutions of equation (0.2). As a consequence, we establish conditions for existence of solutions to (0.2) for any $m,\ 1 \leq m \leq n$, and $K = 1$.

The presented results are obtained jointly with João Lucas M. Barbosa and Jorge Lira from University of Cearà, Brazil and YanYan Li from Rutgers University, USA.

An Attempt to Model Biological Reactions and Deduce an Unknown Mechanism

Seth F. Oppenheimer

Department of Mathematics and Statistics

Mississippi State University

Abstract

We attempted to model the detoxification of a variety of organophosphorus neurotoxicants (OP's) by carboxylesterase (CbxE) in rat serum. As it is known that the chemical reaction is stoichiometric with one molecule of OP detoxified by one molecule of CbxE, we expected a quick fitting to a straight forward mass action type model would neatly tie up the problem of modeling our data and allow us to move on to integrating the model into a larger system. Unfortunately, the behavior of data and more detailed descriptions of the experimental results led us to suspect another mechanism was also affecting solution OP concentration. We have proposed a sorption-like action, such as uptake by blood lipids, as the complicating factor.

We are currently working with our data to see if this explanation, when modeled in a system of ordinary differential equations, will

1. Explain the results we see with a single OP in serum.
2. Will predict what happens when there are binary mixtures of OP's in serum

This is very much a work in progress at the writing of this abstract but our preliminary results are promising.

The experimental work was done in the laboratory of Howard Chambers, Department of Entomology and Plant Pathology, Mississippi State University as part of a larger project with collaborators Janice Chambers and Russell Carr, Center for Environmental Health Sciences, Mississippi State University under a contract with The American Chemistry Council, Contract #0161.

An existence result for a class of p-Laplacian semipositone equations

Shobha Oruganti (speaker)

Mathematics, Penn State Erie, The Behrend College

Erie, PA 16563

Ratnasingham Shivaji

Department of Mathematics, Mississippi State University

Mississippi State, MS 39762.

Abstract

We study positive $C^1(\bar{\Omega})$ solutions to classes of boundary value problems of the form

$$\begin{aligned} -\Delta_p u &= \lambda f(u) - c \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z := \text{div}(|\nabla z|^{p-2}\nabla z)$; $p > 1$, $\lambda > 0$, $c > 0$ are parameters, Ω is a bounded domain in R^N; $N \geq 2$ with $\partial\Omega$ of class C^2 and connected (if $N = 1$, we assume that Ω is a bounded open interval) and f is a $C^1([0, \infty))$ function such that $f(0) = 0$, $f(u) > 0$ for $0 < u < r$ and $f(u) \leq 0$ for $u \geq r$. We establish positive constants $c_0(\Omega, r)$ and $\lambda^*(\Omega, r, c)$ such that the above equation has a positive solution when $c \leq c_0$ and $\lambda \geq \lambda^*$. Our proof is based on sub-super solution techniques.

Finite Time Extinction of a Functional Diffusion Absorption Equation

Mark E. Oxley

Mathematics and Statistics, Air Force Institute of Technology

Wright Patterson AFB, OH 45433 7765

Abstract

This talk will discuss a diffusion absorption equation on a bounded domain with linear diffusion but with a nonlinear absorption term that has a functional component that depends on the concentration function. Certain behavior of this functional will produce global extinction of the concentration in finite time. That is, a positive time such that the concentration is zero then on the entire domain. We discuss necessary and sufficient conditions on the functional term that yields extinction in finite time.

On Oscillatory Third Order Delay Differential Equations

Seshadev Padhi

Department of Mathematics

Jagannath Institute for Technology and Management

Paralakhemundi - 761211, Orissa, India

Abstract

Sufficient conditions in terms of the coefficient functions have been obtained

so that all oscillatory solutions of the third order delay differential equation

$$\left(r_2(t)\left(r_1(t)y'(t)\right)'\right)' + p(t)f(y(g(t))) = F(t)$$

tend to zero as t tends to infinity .

1991 AMS Subject Classification: 34 k 15

Key Words: Oscillation, asymptotic behaviour

Periodic solutions of second order differential inclusions

Daniel Pasca

Mathematical Sciences Department

Worcester Polytechnic Institute

Worcester, MA 01609

Abstract

Using an abstract framework due to Clarke we prove the existence of periodic solutions for second-order differential inclusions systems. For this we use the variational methods (direct method, nonsmooth variant of minimax theorems).

Stability In Neutral Nonlinear Differential Equations With Functional Delays Using Fixed Point Theory

Youssef N. Raffoul

Department of Mathematics

University of Dayton

Dayton, OH 45469 2316

Abstract

We use fixed point theorems to obtain stability results about the trivial solution of the nonlinear neutral differential equation with functional delay

$$x'(t) = -a(t)x(t) + c(t)x'(t - g(t)) + q\left(x(t.)x(t - g(t)\right)$$

Examples will be given to illustrate our theory.

Multiple Positive Solutions for Classes of p-Laplacian Equations

Mythily Ramaswamy
TATA Institute for Fundamental Research Centre IISc Campus,
Bangalore - 560012, India
Ratnasingham Shivaji (speaker)
Department of Mathematics,
Mississippi State University
Mississippi State, MS 39762.

Abstract

We study positive $C^1(\bar{\Omega})$ solutions to classes of boundary value problems of the form

$$\begin{aligned} -\Delta_p u &= \lambda f(u) \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \end{aligned}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z := \text{div}(|\nabla z|^{p-2}\nabla z)$; $p > 1$, $\lambda > 0$ is a parameter and Ω is a bounded domain in R^N; $N \geq 2$ with $\partial\Omega$ of class C^2 and connected. (If $N = 1$, we assume that Ω is a bounded open interval.) In particular, we establish existence of three positive solutions for classes of nondecreasing, p-sublinear functions f belonging to $C^1([0,\infty))$. Our proofs are based on sub-super solution techniques.

Regularity of Neumann Solutions to an Elliptic Free Boundary Problem

Sarah Raynor
University of Toronto
Fields Institute forResearch in the Mathematical Sciences
222 College Street, Toronto, ON Canada
sraynor@fields.utoronto.ca.

Abstract

In this talk, I will examine the regularity properties of solutions to an elliptic free boundary problem. Let $\Omega \subset \mathbb{R}^n$ be a bounded, convex domain. Let $S \subsetneq \partial\Omega$, and let u be the minimizer of

$$J[v] = \int_\Omega \left(|\nabla u|^2 + Q^2(x)\chi_{\{u>0\}}\right)$$

in $K = \{v \in H^1 : v \geq 0 \text{ and } v = u_0 \text{ on } S\}$. Then, u is nonnegative and harmonic in the positive phase $\{u > 0\}$. In addition, $|\nabla u|$ will have a jump of size $Q(x)$ across the free boundary $\partial\{u > 0\}$. Moreover, in this variational formulation, u naturally satisfies Neumann boundary conditions, $\frac{\partial u}{\partial \nu} = 0$, on $\Gamma = \partial\Omega \setminus S$. I prove that such functions are Lipschitz on any compact subset of $\Omega \cup \Gamma.$, so u is Lipschitz up to Neumann boundary segments. I will also demonstrate partial results towards the goal of Lipschitz continuity in the two-phase case, i.e. when u is not required to be non-negative.

Numerical Approximation of Fractional Advection Diffusion Equations.

John Paul Roop

Clemson University

Abstract

In this talk, we introduce the fractional advection dispersion equation, and discuss how it generalizes the ordinary advection dispersion equation to include a dispersive term involving a fractional differential operator. We then present analysis behind a Galerkin formulation of time independent and time dependent fractional advection dispersion equation. Existence and uniqueness results are presented for the time independent and time dependent problems, as well as error estimates. Numerical results are included which confirm the theoretical estimates.

Semiclassical Wave-packet Scattering in One and Two Dimensions

Ivan Rothstein

Virginia Tech

Abstract

In this talk I plan to discuss the contents of a recently submitted article Semiclassical Wave-packet scattering in one and two dimensions. Semiclassical Wave-Packets are a construction used to relate the classical mechanics of a system to the quantum mechanics of a system. For the two body problem we construct scattering theory for the semiclassical wave-packets and then prove that our construction is accurate as $\hbar \to 0$. The result is an extension of an old result by G. Hagedorn to one and two dimensions. I will discuss the shortcomings of the old result in one and two dimensions and how we overcame these shortcomings.

Preliminary Study of a van der Pol Type Oscillator Equation

Sandra Rucker (speaker) and Ronald Mickens
Clark Atlanta University
Atlanta, GA 30314, USA

Abstract

A “highly" nonlinear van der Pol type oscillator differential equation is introduced and the fundamental properties of its solutions are determined. This equation is

$$\ddot{x} + x^{5/3} = \epsilon(1 - x^{2/3})\dot{x},$$

where ϵ is a positive parameter. We show that it has a unique limit-cycle which is stable. From a global point of view, all initial conditions lead to trajectories approaching the limit-cycle as $t \to \infty$. The method of harmonic balance is used to estimate the parameters (amplitude and period) of the limit-cycle.

*Research supported in part by grants from DOE and the MBRS-SCORE Program at Clark Atlanta University.

Bessel functions and generalized eigenfunction expansions

Gary Russell
UAB

Abstract

We relate Bessel function expansions to the theory of generalized eigenfunction expansions.

To persist or not to persist?

Sebastian J. Schreiber
Department of Mathematics
College of William and Mary
Williamsburg, Virginia 23187-8795

Abstract

Kolmogorov vector fields $\dot{x}_i = x_i f_i(x)$ on the non-negative cone $\mathbf{R}^n_+$ on $\mathbf{R}^n$ are often used to describe the dynamics of n interacting species. These vector fields are called permanent if the boundary $\partial\mathbf{R}^n_+$ of the non-negative cone is repelling and are called robustly permanent if they remain permanent under small perturbations of the vector field. I will discuss an open class of vector fields in which there is a dense permanent subset and a dense subset with attractors

on $\partial \mathbf{R}^n_+$. In particular, this class of vector fields provides a counterexample to Jansen and Sigmund's conjecture that permanent vector fields can be generically approximated by robustly permanent vector fields. This presentation will be based on joint work with Josef Hofbauer (Vienna).

Anti Maximum Principle and Bifurcation

Junping shi

Department of Mathematics,

College of William and Mary,

Williamsburg, VA 23187 8795

Abstract

We will discuss two problems related to anti maximum principle. First a new approach to anti maximum principle will be introduced, and we will discuss the ralation between maximum principle, anti maximum principle and a bifurcation problem; second we will discuss how anti maximum principle can hold when the parameter lambda is much larger than the principal eigenvalue. We will also discuss applications of anti maximum principle to a fishery management problem if time allows.

Nonlocal Navier-Stokes Equations:Analytical Results

Valentino A. Simpao

Mathematical Consultant Services

108 Hopkinsville St.

Greenville,KY 42345 USA

Abstract

Auxiliary condition problems for a nonlocal variant of the Navier Stokes equations is considered within the framework of formal Heaviside methods, resulting in analytical solutions for the system. Physically, such results model fluid dynamics with nonlocal convective flows[e.g., plasma dynamics of stars].

Uniqueness of positive radial solutions for $\Delta u + f(u) = 0$ on the annulus

Zachariah Sinkala

Department of Mathematical Sciences

Middle Tennessee State University

Murfreesboro, Tennessee 37132

Abstract

We prove uniqueness of positive radial solutions to the semilinear elliptic equations $\Delta u + f(u) = 0$, subject to the Dirichlet boundary condition on an annulus in $\mathbf{R}^n$, $n \geq 0$. The proofs given here have different approach to work done by others on the similar topic.

Competitive dynamics in a model for river blindness with cross-immunity

Horst Thieme (Invited Speaker)

Arizona State University
(Joint work with Jimmy Mopecha based on his Ph.D. dissertation)

Abstract

A mathematical model is formulated for onchocerciasis (river blindness) which involves both the parasites (threadlike worms) afflicting humans (onchocerca volvulus) and cattle (onchocerca ochengi). Both parasites are transmitted by the same flies. Since the flies bite both humans and cattle, every now and then they transmit the parasites into the wrong host where the parasites soon die without causing harm. There is circumstantial evidence that the parasites trigger immune reactions even in the wrong hosts which protects against the other parasite. The model solutions show the phenomena which are well-known known from the classic Volterra-Lotka two-species competition model. Using persistence theory and monotone dynamical systems with a non-standard order, conditions are determined for competitive exclusion, coexistence, and bistability. It is analyzed how changing the amount of cattle would lead to shifts between these scenarios.

Sensitivity Equations for the Design of Control Systems

James Vance

Department of Mathematics

Virginia Tech

Abstract

Systematic strategies for optimal actuator and sensor locations require finding extrema of control performance measures. When the control is designed for a distributed parameter system, these performance measures frequently involve the kernel of the Riccati operator or that of the feedback operator. For example, the optimal linear quadratic regulator cost over a range of initial data involves the Riccati operator. We consider sensitivity equations for Riccati and Chandrasekhar equations. The latter is well-suited for computing feedback kernels when there is a small number of control inputs and control outputs. As we shall see, the sensitivity of these kernels can lead to efficient computation of gradients for optimization algorithms and produce valuable information independently. Numerical examples corresponding to placing actuators in the heat equation are provided.

Mathematical Analysis of the Global Dynamics of a Model for HIV Infection of CD4^+ T cells

Liancheng Wang

Department of Mathematical Science

Georgia Southern University

Abstract

A mathematical model that describes the interaction of HIV infection and CD4$^+$ T cells is studied. Global dynamics analysis is carried out. By identifying a basic reproduction number R_0, it is proved that, if $R_0 \leq 1$, the disease dies out and no HIV infection persists. If $R_0 > 1$, the HIV infection survives and it persists, solutions approaching either the unique endemically infected steady state or a periodic orbit.

The inverse problem for a finite tree

Rudi Weikard
Department of Mathematics
University of Alabama at Birmingham
Birmingham, AL 35226

Abstract

We prove that the Dirichlet to Neumann map of a finite simply connected tree determines uniquely the potential on the tree.

Local Regularity Theorems for the Stationary Thermistor Problem

Xiangsheng Xu
Professor of Mathematics
Department of Mathematics and Statistics
Mississippi State University
Mississippi State
MS 39762

Abstract

In this paper we present a simplier proof of a result of Lewis [L] concerning the continuity of weak solutions to the two dimensional thermistor problem in the case where the temperature can blow up in a region with non empty inteior. Some other regularity properties are also discussed.

Local Regularity Theorems for the Stationary Thermistor Problem

Xiangsheng Xu
Professor of Mathematics
Department of Mathematics and Statistics
Mississippi State University
Mississippi State
MS 39762

Abstract

In this paper we present a simplier proof of a result of Lewis [L] concerning the continuity of weak solutions to the two dimensional thermistor problem in the case where the temperature can blow up in a region with non empty inteior. Some other regularity properties are also discussed.

Inverse medium scattering problem in layered half-space

Yongzhi Xu
Department of Mathematics
University of Tennessee at Chattanooga
yxu@cecasun.utc.edu

Abstract

This talk concerns with the inverse medium scattering problem in a perturbed layered half-space. A wave penetrable object is located in a layer where the refraction index is different from the other part of the half space. Wave propagation in such a layered half space is different from that in a homogeneous half space.

In a layered half space, scattered wave consists of free wave and guided wave. In many cases only free-wave far-field or only guided-wave far-field can be measured. Our research concerns the problems of determining the wave penetrable object from incomplete far-field data.

We establish mathematical formulas for relations of the object, incident wave, scattered near field and far field. In the ideal condition when complete data are given, we prove uniqueness of the problem.

From computational aspect we develop a dual space indicator method to approximate the unknown object from measured near field or far field data.

Variational Method for Groundwater Modeling

Ian Knowles and Aimin Yan (speaker)
Department of Mathematics
University of Alabama at Birmingham

Abstract

In order to recover the coefficients in groundwater modeling equations, we construct a convex functional which has a global minimum at the recovered point. By minimize the functional, we can recover all the coefficients. This method is proven to be effective.

On the Stationary Solution of the Mathematical Model for Grain Boundary Grooving

Akira Yanagiya
Advanced Institute for Complex Systems
Waseda University

Abstract

In this talk we will present the application of admissibility of integral operator for the mathematical model of the grain grooving on the policrystal.

Large Positive Solutions of a Third Order Boundary Value Problem

John R. Graef (john-graef@utc.edu),
Department of Mathematics,
University of Tennessee at Chattanooga,
Chattanooga, TN 37043

Chuanxi Qian (qian@math.msstate.edu),
Department of Mathematics and Statistics,
Mississippi State University,
Mississippi State, MS 39762

Bo Yang (speaker) (byang@kennesaw.edu),
Department of Mathematics,
Kennesaw State University,
Kennesaw, GA 30144

Abstract

In this paper, the authors consider the boundary value problem

$$u'''(t) = \lambda g(t) f(u),\ 0 < t < 1, \qquad (E)$$

$$u(0) = u'(p) = u''(1) = 0, \qquad (B)$$

where $\lambda > 0$ is a parameter, $p \in (\frac{1}{2}, 1)$ is a constant. Some existence and nonexistence results for positive solutions of the problem (E)-(B) are obtained.

Contraction Mapping and Stability in a Delay-Differential Equation

Bo Zhang
Department of Mathematics and Computer Science
Fayetteville State University
Fayetteville, NC 28301 USA

Abstract

In this paper we study the stability properties of a delay differential equation by means of contraction mappings. The paper is motivated by a number difficulties encountered in the study of stability by means of Liapunov's direct method. We notice that most of these difficulties vanish when applying fixed point theory. An asymptotic stability theorem with a necessary and sufficient condition is proved.

Differential Operators and Weakly Stable Manifolds

Hongkun Zhang
Department of Mathematics
University of Alabama at Birmingham
452 Campbell Hall
Birmingham AL 35294 1170

Abstract

In the last three decades of the twentieth century, chaotic billiards became one of the most active and popular research areas in dynamical systems. The ergodicity of billiards would lead to a solution, in some form, of the famous Boltzmann's ergodic hypothesis on gases of hard balls. To prove the ergodicity of billiard flow on m dimensional Riemannian manifold with boundary is still an open problem. Fundamental to the study of ergodic theory is the construction of a submanifold of unit sphere bundle called the weakly stable manifold. In billiards dynamics, this construction is somewhat difficult. In my recent joint work with Robert Kauffman, we give an alternative approach using differential operator theory.

Harnack type inequalities for some nonlinear elliptic equations on manifolds

Lei Zhang
Texas A&M University
lzhang@math.tamu.edu

Abstract

Let (M, g) be a smooth Riemannian Manifold and let $B(p, 1)$ be a geodesic ball centered at p, we prove Harnack type inequalities for the famous Yamabe equation defined over $B(p, 1)$. These inequalities will lead to the energy bound of the solutions of the Yamabe equation and a sharp estimate near a blow-up point.