Math 338 Final Exam Fall 2008

100 points total Name: _____

All computations should be computed using Matlab. Any commands that you use as well as the output should be included in a word document named yourname_final.doc. All m-files that you write yourself or modify for the test should be turned in.

1. The seven point Newton-Cote quadrature method has the form

$$Q_7(f,a,b) = \sum_{i=1}^{7} a_i f(x_i) \approx \int_{-1}^{1} f(x)dx$$

   a. What are the values of the weights a_i? Use **format rat** while finding the weights.

   b. If f(x) is a polynomial then what is the highest degree of a polynomial for which $Q_7(f,a,b)$ will calculate $\int_{-1}^{1} f(x)dx$ exactly?

2. Each of the following matrices is the augmented form $[A \mid b]$ of a corresponding system of equations $Ax = b$ (x and b are column vectors).

   Determine if each of the systems of equations is consistent or inconsistent using the **rref** command. If a system is consistent then either give a unique solution for x or give a general solution for x which will represent all of its solutions.

   a. $\left[\begin{array}{cc|c} 2 & -1 & 2 \\ 1 & 1 & 7 \\ -3 & 2 & -1 \end{array}\right]$

   b. $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -1 & 1 & 5 & 1 \\ 2 & -1 & -7 & -1 \end{array}\right]$

   c. $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -1 \\ 2 & 2 & -1 & 3 & 4 \\ -1 & -1 & -2 & 1 & 4 \end{array}\right]$

3. Consider the following data:

x	1	2	3	4
y	8	12	16	21

We would like to fit this data with a function of the form $F(x) = \frac{c_1}{x} + c_2 x$ in a least squares sense.

   a. Set up a system of equations $Ac = y$ where $c = (c_1, c_2)^T$ such that the backslash command in Matlab can find the best choice of c. What is A? What is y?

   b. Find c using backslash with the normal equations.

   c. Find c using polyfit.

   d. Plot the function F(x) over the time interval [0.2,5] using a stepsize of h = 0.2. On the same plot show the original data using circles that are not connected by any lines. Give the plot a title of "Plot: nonlinear data fit".

4. The following m-file gauss4quad is supposed to approximate the integral $\int_a^b f(x)dx$ using a 4-point Gaussian quadrature method.

```
function intvalue = gauss4quad(funname, a, b)
% gauss4quad.m
% a 4 point gaussian quadrature rule for approximating
% int(f, a, b)

ti = gnodes(4);
ai = gweights(4);

fi = feval(funname, ti);

intvalue = sum(ai.*fi);
```

Unfortunately, gauss4quad is coded incorrectly so that it will only give approximations of the integral $\int_{-1}^{1} f(x)dx$.

   a. Fix this m-file so that it will correctly approximate the integral $\int_a^b f(x)dx$. Call this fixed m-file mygauss4quad.

   b. Use mygauss4quad to approximate the integral $\int_0^5 xe^{-\frac{x}{2}}dx$. What is the exact value of this integral? What is the error in using mygauss4quad? (Hint: use format long and double as needed).

5. The following method attributed to Ralston (1962) is a three-stage Runge-Kutta method (i.e. is uses three slope calculations) for approximating the solution of the IVP $\begin{matrix} y' = f(t.y) \\ y(t_0) = y_0 \end{matrix}$

Ralston Method

$$y_i = y_{i-1} + h\left(\frac{2}{9}s_1 + \frac{1}{3}s_2 + \frac{4}{9}s_3\right)$$

where

$s_1 = f(t_{i-1}, y_{i-1})$

$s_2 = f(t_{i-1} + (h/2), y_{i-1} + (h/2)s_1)$

$s_3 = f(t_{i-1} + (3h/4), y_{i-1} + (3h/4)s_2)$

a. Write an m-file myralston which will calculate the numerical solution of the IVP, $\begin{matrix} y' = f(t.y) \\ y(t_0) = y_0 \end{matrix}$ on the interval $\left[t_0, t_f\right]$ with step-size of h. The m-file should be called with the command

>> [t,y] = myralston(funname, y0, t0, tf, h)

(Hint: you may want to modify the file myheun).

b. Calculate the numerical solution for the initial value problem $\begin{matrix} y' = y - t \\ y(1) = 1 \end{matrix}$ on the interval [1,2] using step-size of h = 0.2. What is the approximate value of y(2) that you computed?

6. Consider the IVP

$u'' + 2u' + 3u = 0$

$u(0) = 2$

$u'(0) = 0$

a. Write this IVP as a system of first order differential equations $\vec{y}' = f(t, \vec{y})$ with initial condition $\vec{y}(0) = \vec{y}_0$ where $\vec{y}$ is a 2×1 column vector. Be sure to specify what the initial condition is.

b. Use pplane7 to plot the solution. Be sure to use *keyboard input* under the *solutions* menu to specify the initial conditions.