Math 311 **Graded Homework 3**
Fall 2008 Name:__________________
35 points total

1) Solve using variation of parameters: $y''' + y' = \csc(x)$.

2) Solve using undetermined coefficients: $y''' - y'' - 4y' + 4y = e^{-x}$.

3) Determine the largest t-interval in which solutions are guaranteed to exist for

$$(t-1)y^{(4)} + \ln(t)y'' = t-1$$
$$y(1/2) = 1$$

4) Show that $\{t, t^2, t^4\}$ form a fundamental set of solutions for

$$t^3 y''' - 4t^2 y'' + 8ty' - 8y = 0.$$

5) Write $y'' + t^2 y' + 4y = \sin(t)$ as a system of 1st order linear differential equations.

6) Find the eigenvalues and eigenvectors for $\begin{pmatrix} 1 & -3 \\ 1 & 5 \end{pmatrix}$

7) Find the general solution for the system $\begin{cases} y_1' = y_1 + y_2 \\ y_2' = 4y_1 + y_2 \end{cases}$ when

$$y_1(0) = 1,\ y_2(0) = 0$$

8) Find the general solution for $\vec{y}' = \begin{pmatrix} 1 & -3 \\ 1 & 5 \end{pmatrix} \vec{y}$.

9) Find the general solution for $\vec{x}\,' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \vec{x}$

10) Find the general solution for $\vec{x}\,' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \vec{x}$

11) Let α be a parameter in the system $\vec{x}\,' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \vec{x}$

a. Determine the eigenvalues in terms of α .
b. Find the critical value(s) of α where the qualitative nature of the phase portrait for the system changes.
c. Draw a phase portrait for a value of α slightly below, and for another value slightly above, each critical value.

12) Draw example phase portraits for the following:
a. Saddle point
b. Stable node
c. Unstable node
d. Stable spiral
e. Unstable spiral
f. Center
g. Improper node