

## Exercises #8

1. Let  $\mathbf{x} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ . Find  $\|\mathbf{x}\|_1$ ,  $\|\mathbf{x}\|_2$  and  $\|\mathbf{x}\|_\infty$ .
2. Let  $A = \begin{pmatrix} 3 & 5 \\ -1 & 2 \end{pmatrix}$ . Find  $\|A\|_1$ ,  $\|A\|_2$  and  $\|A\|_\infty$ .
3. Consider the system  $A\mathbf{x} = \mathbf{b}$  where  $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ . The exact solution of this system of equations is  $\mathbf{x}^* = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ .
  - (a) Verify that  $\mathbf{x}^*$  is the exact solution by computing the norm of  $\mathbf{e} = A\mathbf{x} - \mathbf{b}$  using the  $l_1$  norm, the  $l_2$  norm and the  $l_\infty$  norm.
  - (b) The following iteration is derived from a method called the Jacobi method which can be used to solve the above system of equations.

$$\mathbf{x}^{(k)} = \begin{pmatrix} 0 & -1/3 \\ 1/2 & 0 \end{pmatrix} \mathbf{x}^{(k-1)} + \begin{pmatrix} 2/3 \\ -3/2 \end{pmatrix}$$

Find  $\mathbf{x}^{(k)}$  for  $k = 1, 2, 3$ . For each vector, calculate the relative and absolute errors using the  $l_\infty$  norm.

$$\text{absolute error} = \mathbf{e}_{\text{abs}}^{(k)} = \|\mathbf{x}^{(k)} - \mathbf{x}^*\|$$

$$\text{relative error} = \mathbf{e}_{\text{rel}}^{(k)} = \frac{\|\mathbf{x}^{(k)} - \mathbf{x}^*\|}{\|\mathbf{x}^*\|}$$

Is this method converging to the solution of the system?

4. Let  $\mathbf{x}$  be a vector in  $R^2$ . What do the following plots look like?
  - (a) all vectors  $\mathbf{x}$  where  $\|\mathbf{x}\|_1=1$
  - (b) all vectors  $\mathbf{x}$  where  $\|\mathbf{x}\|_2=1$
  - (c) all vectors  $\mathbf{x}$  where  $\|\mathbf{x}\|_\infty=1$