Homework

Use format long.

1. Use myeuler to compute the solution of the initial value problem $\begin{matrix} y' = y \\ y(0) = 1 \end{matrix}$ over the interval [0,1] using a step-size of h = 0.1.

2. The exact solution to problem 1 is $y = e^x$. How does your answer at t = 1 compare with the exact answer?

3. Write an m-file called myheun.m which will implement Heun's Method.

4. Consider the 1st order D.E. $y' = y - t$. Write an appropriate m-file that can be used to calculate f(t,y).

5. If the D.E. in problem 5 has an initial condition y(1) = 1, then the exact solution is $y = 1 + t - e^{t-1}$.

   a. What is the exact value of y(2)?
   b. Use Heun's Method to approximate y(2) using different values of h where $h = [0.5\ 0.25\ 0.2\ 0.1\ 0.05\ 0.02\ 0.01\ 0.005]$. Display the stepsize, exact value, approximated values and errors in a matrix. Also, use polyfit to find the value of r the errors, e, and the stepsizes, h, obey the relationship $e = Ch^r$. Use the following loop (in heunloop.m) with the appropriate lines filed in.

```
% this is heunloop.m
% This will give an approximation of y(tf) for
% the IVP: y'=f(t,y) with y(t0) = y0.
%
rhsfunname = ...    % function name, a string
t0 = ...            % initial time
tf = ...            % final time

h=[0.5 0.25 0.2 0.1 0.05 0.02 0.01 0.005];

error = zeros(size(h));
n = length(h);

yexact = ...        % exact value of y(tf)

for i = 1:n

    [t,y]=myheun(...)
    error(i) = abs(y(end) - yexact)

end

fprintf('\nStepsize\t\t Exact value \t\t Approximate \t\t Error\n')
fprintf('%4.3f \t %11.10f \t %11.10f \t %11.10e \n',...)
coef = polyfit(...);
fprintf('\nThe order of the Heun Method is h^{%4.3}\n',coef(1))
```

c. Modify heunloop so that is runs the Euler method and call it eulerloop. How do the results compare? What is the order of the Euler Method?