Homework

Use format long.

1. Rewrite myheun so that it implements the classical 4^{th} order Runge-Kutta Method and call it myrk4. In other words, the error should obey the relationship $e = Ch^r$ with r = 4.

   >>[t, y] = myrk4(funname, y0, t0, tf, h)

   Rewrite heumloop as rk4loop and use it to show that the Runge-Kutta method is actually a method of order 4. Use the D.E. $y' = y - t$ with y(1) = 1.

2. A given tank of water initially has 100 gallons of water in it and 15 lbs of salt. A salt solution with a concentration of 0.02 lbs of salt per gallon is fed into the tank at 4 gallons per minute. The well-stirred mixture in the tank is also allowed to exit at the same rate of 4 gallons per minute.
   The amount of salt in the tank, y(t), in pounds, obeys the initial value problem

   $$y' = 0.08 - 0.04y$$
   $$y(0) = 15$$

   a. Use Euler's Method, Heun's Method and the 4^{th} order Runge-Kutta methods to estimate the amount of salt that will be in the tank after 20 minutes using h = 0.5.
   b. How do your answers in part (a) compare with the exact amount of salt at t = 20 if the solution of the IVP is $y = 2 + 13e^{-0.04t}$?
   c. Give three separate plots each comparing one of the numerical methods against the exact solution on the interval [0,20]. Represent the exact solution with a green curve and the approximation with a red curve. Each plot should have an appropriate title indicating the method which was used.

3. Write an m-file called myheunsys, which will run Heun's method for a first order system of m ODE's of the form $\begin{matrix} x' = f(t,x) \\ x(0) = x_0 \end{matrix}$.
   Note: this means that x and f are $m \times 1$ vectors.
   The function should be called in the following way:
   >>[t,x] = myheunsys(rhsfunname, x0, t0, tf, h)
   Where rshfunname will be a function m-file that will return a column vector with m rows, x0 should also be an $m \times 1$ vector.

4. Test myheunsys on the system of ODE's $x' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x$. Use the initial conditions of $x(0) = (1,0)^T$ and solve for times in the interval $[0, 2\pi]$ with h = 0.1.

5. Use pplane7 to plot the same system as the previous problem. Turn in the plot.

6. Use myheunsys and pplane7 to plot solutions of the system $\begin{array}{l} x' = x(1-y) \\ y' = y(x-1) \end{array}$. Use a starting point of $(x_0, y_0) = (1, 1.4)$.