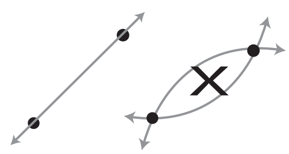
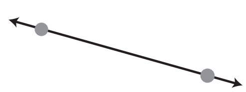
Axioms for Euclidean Geometry

Axioms of Incidence

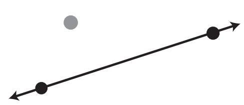
1. For every two points A and B, there exists a *unique* line ℓ that contains both of them.



2. There are at least two points on any line.

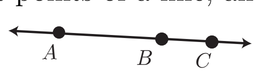


3. There exist at least three points that do not all lie on a line.



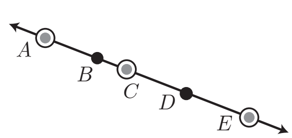
Axioms of Order

1. If $A * B * C$, then the points A, B, C are three distinct points of a line, and $C * B * A$.

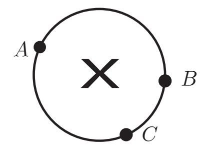


2. For two points B and D, there are points A, C, and E, such that

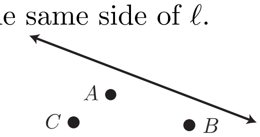
$$A * B * D \quad B * C * D \quad B * D * E$$



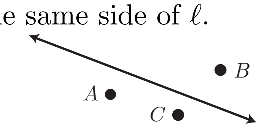
3. Of any three points on a line, there exists no more than one that lies between the other two.



4. (Plane Separation Postulate) For every line ℓ and points A, B, and C not on ℓ:
(i) If A and B are on the same side of ℓ and B and C are on the same side of ℓ, then A and C are on the same side of ℓ.

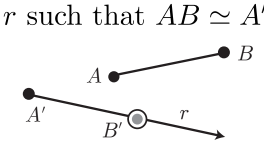


(ii) If A and B are on opposite sides of ℓ and B and C are on opposite sides of ℓ, then A and C are on the same side of ℓ.



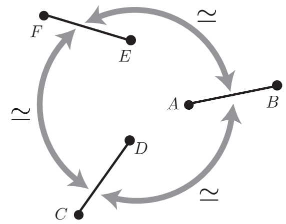
Axioms of Congruence

1. (Segment construction) If A and B are distinct poionts and if A' is any point, then for each ray r emanating from A', there is a unique point B' on r such that $AB \simeq A'B'$.

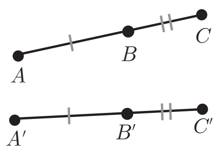


2. If $AB \simeq CD$ and $AB \simeq EF$, then $CD \simeq$

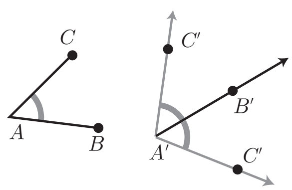
EF. Every segment is congruent to itself.



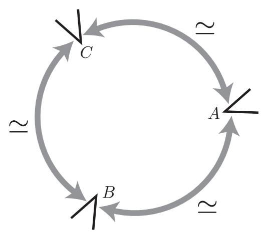
3. (Segment addition) If $A * B * C$ and $A' * B' * C'$, and if $AB \simeq A'B'$ and $BC \simeq B'C'$, then $AC \simeq A'C'$.



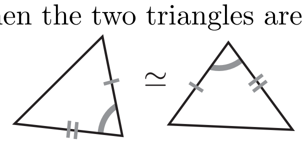
4. (Angle construction) Given $\angle BAC$ and any ray $\overrightarrow{A'B'}$, there is a unique ray $\overrightarrow{A'C'}$ on a given side of $\overline{A'B'}$ such that $\angle BAC \simeq \angle B'A'C'$.



5. If $\angle A \simeq \angle B$ and $\angle A \simeq \angle C$, then $\angle B \simeq \angle C$. Every angle is congruent to itself.

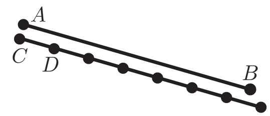


6. (SAS) If two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of another triangle, then the two triangles are congruent.

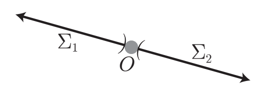


Axioms of Continuity

1. (Archimedes' axiom) If AB and CD are any two segments, there is some number n such that n copies of CD constructed contiguously from A along the ray $\overrightarrow{AB}$ will pass beyond B.



2. (Dedekind's axiom) Suppose that all points on line ℓ are the union of two nonempty set $\Sigma_1 \cup \Sigma_2$ such that no point of Σ_1 is between two points of Σ_2 and vice versa. Then there is a unique point O on ℓ such that $P_1 * O * P_2$ for any point $P_1 \in \Sigma_1$ and $P_2 \in \Sigma_2$.



The Axiom on Parallels

1. (Playfair's postulate) For any line ℓ and point P not on ℓ, there is exactly one line through P parallel to ℓ.