6. READER’S SOLO

SHORTER AND LONGER

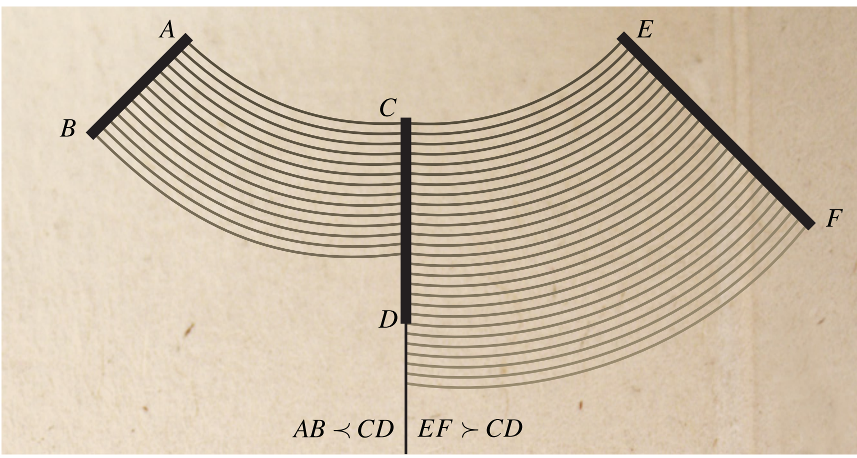
The purpose of this short section is to develop a system of comparison for segments that aren't congruent. I am going to let you provide all the proofs in this section. It will give you the opportunity to work with order and congruence on your own.

DEF: SHORTER AND LONGER

Given segments AB and CD, label E on $CD\rightarrow$ so that $CE \simeq AB$.

If $C * E * D$, then AB is *shorter than* CD, written $AB \prec CD$.

If $C * D * E$, then AB is *longer than* CD, written $AB \succ CD$.



Note that if you replace CD in this definition with DC, things will change slightly: calculations will be done on the ray $DC\rightarrow$ rather than $CD\rightarrow$. That would seem like it could be problem, since CD and DC are actually the same segment, so your first task in this chapter is to make sure that $\prec$ and $\succ$ are defined the same way, whether you are using CD or DC.

THM: $\prec$ AND $\succ$ ARE WELL DEFINED

Given segments AB and CD, label:

E: the unique point on $CD\rightarrow$ so that $AB \simeq CE$ and

F: the unique point on $DC\rightarrow$ so that $AB \simeq DF$.

Then $C * E * D$ if and only if $D * F * C$.

Here are a bunch of the properties of $\prec$ for you to verify. There are, of course, corresponding properties for $\succ$, but I have left them out to cut down on some of the tedium.

THM: TRANSITIVITY OF $\prec$

If $AB \prec CD$, and $CD \prec EF$, then $AB \prec EF$.
If $AB \prec CD$, and $CD \simeq EF$, then $AB \prec EF$.
If $AB \simeq CD$, and $CD \prec EF$, then $AB \prec EF$.

THM: SYMMETRY BETWEEN $\prec$ AND $\succ$

For any two segments AB and CD, $AB \prec CD$ if and only if $CD \succ AB$.

THM: ORDER (FOUR POINTS) AND $\prec$

If $A * B * C * D$, then $BC \prec AD$.

THM: ADDITIVITY OF $\prec$

Suppose that $A * B * C$ and $A' * B' * C'$. If $AB \prec A'B'$ and $BC \prec B'C'$, then $AC \prec A'C'$.