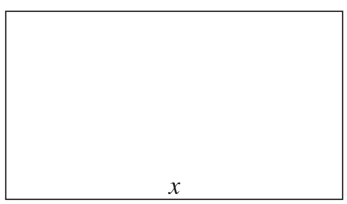
Math 1110 Test 1. Fall 2016

1. (a) Give the formula for the distance d between the points (x_1, y_1) and (x_2, y_2).

(b) Write a function $d(x)$ to describe the distance from $(1, 2)$ to a point (x, y) on the parabola $y = x^2$.

2. (a) A farmer wants to enclose a rectangular plot of land with 200 linear feet of fencing. Write a function $A(x)$ to describe the area of the plot in terms of its length x.



(b) What is the domain of the function (as it relates to this problem)?

3. (a) What is the rate of change of the linear function $f(x) = 5x - 10$?

(b) What is $f(0)$?

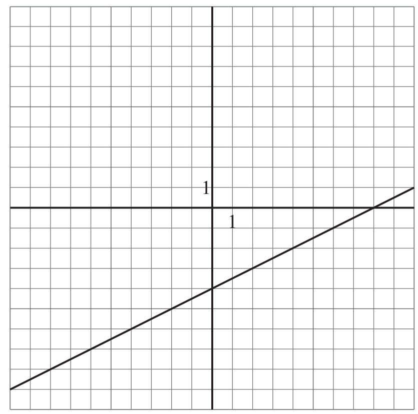
(c) For what value x is $f(x) = 0$?

4. (a) What is the slope of the linear function $f(x)$ that passes through the points $(2, 3)$ and $(4, 7)$?

(b) What is its y-intercept?

(c) What is the equation of the function?

5. The graph of the linear function $y = f(x)$ is shown.

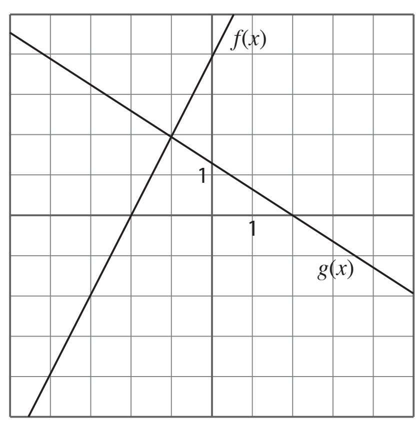


(a) Is it increasing or decreasing?

(b) What is its slope?

(c) What is its y-intercept?

6. The graphs of the functions $f(x)$ and $g(x)$ are shown.



(a) For which x values is $f(x) \geq 0$?

(b) For which x values is $g(x) \geq 0$?

(c) For which x values is $f(x) \geq g(x)$?

7. (a) What is the vertex of

$$f(x) = 2x^2 + 4x + 6?$$

(b) Does the parabola "open up" or "open down"?

(c) What is the y-intercept?

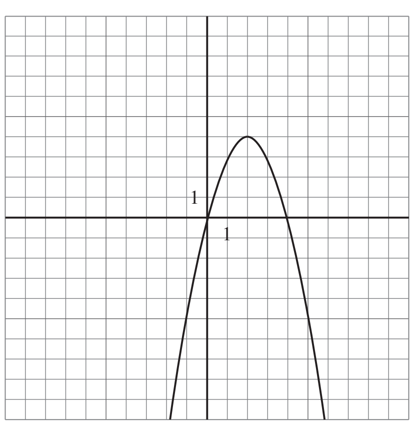
(d) Use the information gathered in (a), (b), and (c) to graph the function. Be sure to label the axes and all relevant points on the graph.

8. Does the quadratic function have any x-intercepts? If yes, what are they? If no, explain why not.

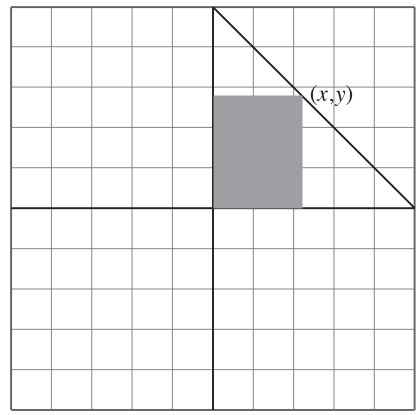
(a) $f(x) = x^2 + 2x + 5$

(b) $f(x) = x^2 + 5x + 2$

9. The graph of a quadratic function $f(x)$ is shown. Write an equation for it in the standard form $f(x) = a(x-h)^2 + k$.



10. A rectangle in the first quadrant has one corner on the line $y = 5 - x$, as shown. What value x will maximize the area of the rectangle?



SOLUTIONS

1. (a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(b)

$$\begin{aligned} d(x) &= \sqrt{(x-1)^2 + (x^2-2)^2} \\ &= \sqrt{x^2 - 2x + 1 + x^4 - 4x^2 + 4} \\ &= \sqrt{x^4 - 3x^2 - 2x + 5} \end{aligned}$$

2. (a) The perimeter is 200, so $2x + 2y = 200 \implies x + y = 100 \implies y = 100 - x$. The area enclosed is

$$A(x) = x(100 - x) = 100x - x^2.$$

(b) $0 < x < 100$

3. (a) The rate of change is the slope, 5.

(b) $f(0)$ is the y-intercept, -10.

(c)

$$\begin{aligned} f(x) &= 0 \\ 5x - 10 &= 0 \\ 5x &= 10 \\ x &= 2. \end{aligned}$$

4. (a)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4-2} = 2.$$

(b) In slope-intercept form, the equation of the line is

$$y = 2x + b.$$

Plug in the point $(2, 3)$ to find b:

$$3 = 2 \cdot 2 + b \implies b = -1.$$

(c) The function is $f(x) = 2x - 1$.

5. (a) It is increasing.

(b) The slope is $1/2$.

(c) The y-intercept is -4.

6. (a) $f(x) \geq 0$ when $x \in [-2, \infty)$.

(b) $g(x) \geq 0$ when $x \in (-\infty, 2]$.

(c) $f(x) \geq g(x)$ when $[-1, \infty)$.

7. (a) Use the vertex formula:

$$h = -\frac{b}{2a} = -\frac{4}{2 \cdot 2} = -1.$$

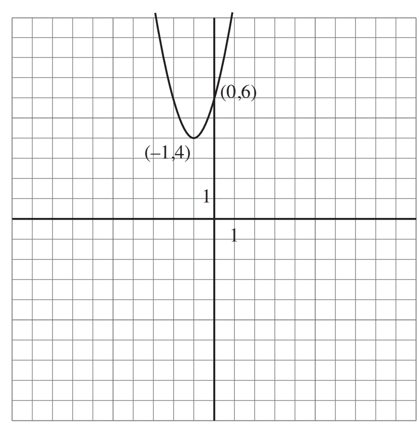
$$k = f(-1) = 2(-1)^2 + 4(-1) + 6 = 4.$$

The vertex is $(-1, 4)$.

(b) The leading coefficient is greater than zero, so the parabola opens up.

(c) $f(0) = 2 \cdot 0^2 + 4 \cdot 0 + 6 = 6$.

(d)



8. (a) Look at the discriminant:

$$\Delta = b^2 - 4ac = 2^2 - 4 \cdot 1 \cdot 5 < 0.$$

Therefore the function has no real zeros.

(b) It does not factor, so use the quadratic formula

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} \\ &= \frac{-5 \pm \sqrt{17}}{2}\end{aligned}$$

9. The vertex is $(2, 4)$, so

$$f(x) = a(x-2)^2 + 4.$$

The parabola passes through the point $(0, 0)$, so

$$0 = a(0-2)^2 + 4 \implies -4 = 4a \implies a = -1.$$

The equation is

$$f(x) = -(x-2)^2 + 4.$$

10. The area of the rectangle is

$$\begin{aligned}A &= x \cdot y \\ &= x(5-x) \\ &= 5x - x^2\end{aligned}$$

It is a quadratic function that opens down, so the maximum occurs at the vertex. Use the vertex formula to find it:

$$h = -\frac{b}{2a} = -\frac{5}{2 \cdot (-1)} = \frac{5}{2}.$$

The maximum area occurs when $x = 5/2$.