Math 1110 Test 2. Fall 2016

1. Solve the inequality: $x^2 - 2x - 8 \leq 0$.

2. Let $f(x) = x^2 + x$ and $g(x) = 5x + 5$. For what values of x is $f(x) \geq g(x)$?

3. Identify which of the functions are polynomials and which are rational functions. Put a check in the box to indicate that it is a polynomial or rational function.

$$f(x) = \frac{1}{x^2}$$
$$f(x) = x(x-3)(x+4)$$
$$f(x) = \frac{|x|+2}{|x|}$$
$$f(x) = \frac{x^2+x}{1-x}$$
$$f(x) = \sqrt{\frac{x^2+1}{x}}$$

4. $f(x) = x^3 - 3x^2$

(a) What is the degree of $f(x)$? What is its leading coefficient? What is its end behavior (describe it in words or draw it)?

(b) Make a table of the zeros of $f(x)$ and their multiplicities.

(c) Use the information from the previous parts to sketch the graph of $f(x)$. Be sure to label all relevant information.

5. $f(x) = x(x-1)(x+2)^2$

(a) What is the degree of $f(x)$? What is its leading coefficient? What is its end behavior (describe it in words or draw it)?

(b) Make a table of the zeros of $f(x)$ and their multiplicities.

(c) Use the information from the previous parts to sketch the graph of $f(x)$. Be sure to label all relevant information.

6. Find the equation of the slant (oblique) asymptote of the rational function

$$f(x) = \frac{x^3 + 5x^2 + 1}{x^2 + 3x}.$$

7. $f(x) = \dfrac{x+2}{x(x-3)(x+1)}$

(a) What is the domain of $f(x)$?

(b) List the vertical asymptote(s) of $f(x)$.

(c) Does $f(x)$ have a horizontal asymptote, and if so, what is it?

(d) What is the y-intercept?

(e) List the x-intercept(s).

(f) Graph $f(x)$. Label all relevant information.

8. $f(x) = \dfrac{x^2+3x-4}{x^2+x-2}$

(a) What is the domain of $f(x)$?

(b) List the vertical asymptote(s) of $f(x)$.

(c) Does $f(x)$ have a horizontal asymptote, and if so, what is it?

(d) What is the y-intercept?

(e) List the x-intercept(s).

(f) Graph $f(x)$. Label all relevant information.

SOLUTIONS

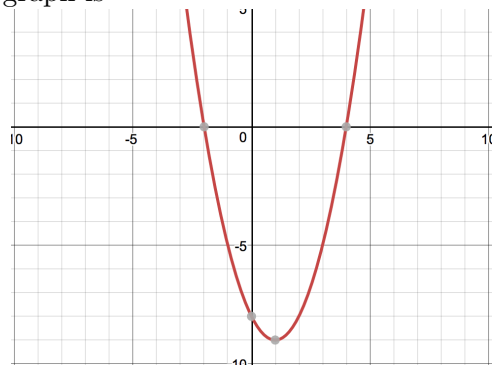
1. The function $f(x) = x^2 - 2x - 8$ is quadratic. Its vertex has coordinates

$$h = -\frac{(-2)}{2(1)} = 1, \quad k = (1)^2 - 2(1) - 8 = -9.$$

Its x-intercepts are:

$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = 4, \quad x = -2$$

Its graph is



so the solution to the inequality is $[-2, 4]$.

2. We need to solve the inequality

$$x^2 + x \geq 5x + 5$$
$$x^2 - 4x - 5 \geq 0$$

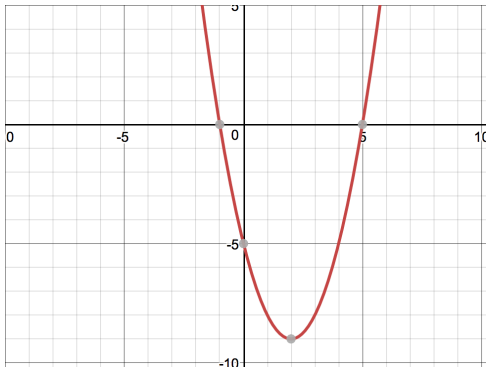
The function $f(x) = x^2 - 4x - 5$ is quadratic. Its vertex has coordinates

$$h = -\frac{(-4)}{2(1)} = 2, \quad k = 2^2 - 4 \cdot 2 - 5 = -9.$$

Its x-intercepts are

$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$$x = 5, \quad x = -1.$$

The graph is



so the solution to the inequality is $(-\infty, -1] \cup [5, \infty)$.

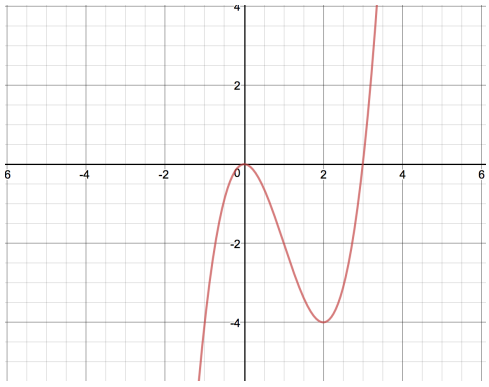
3. Only the second one is a polynomial. The first, second, and fourth are rational functions.

4. (a) The leading coefficient is 1 and the degree is 3, so the function falls to the left and rises to the right.

(b)

zero	mult.
0	2
3	1

(c)

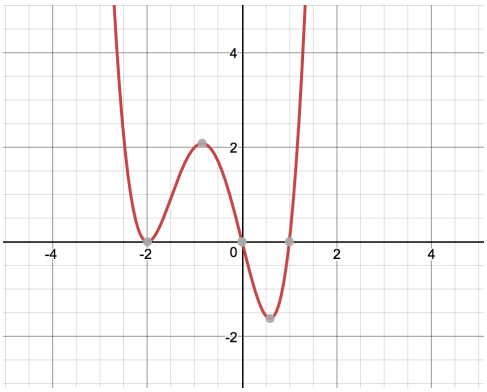


5. (a) The leading coefficient is 1 and the degree is 4, so the function rises on both left and right.

(b)

zero	mult.
0	1
1	1
-2	2

(c)



6. Use long division to divide $x^2 + 3x$ into $x^3 + 5x^2 + 1$. The quotient is $x + 2$ (with remainder $-6x + 1$), so the equation of the slant asymptote is $y = x + 2$.

7. (a) The domain is the set of all real numbers except 0, 3, and -1.

(b) The vertical asymptotes are $x = 0$, $x = 3$ and $x = -1$.

(c) Yes, $y = 0$ (because the degree of the numerator, 1, is less than the degree of the denominator, 3.

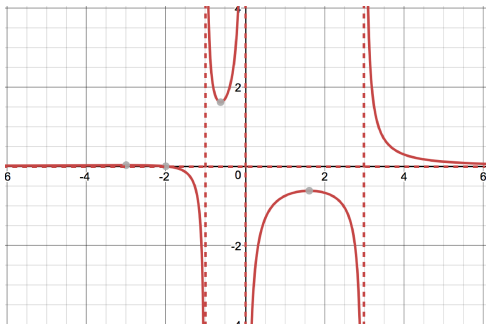
(d) There is no y-intercept (because 0 is not in the domain of the function).

(e) $x = -2$

(f) Note that you need to plot at least one more point. For instance, if $x = -3$,

$$f(-3) = \frac{-3+2}{-3(-6)(-2)} = \frac{1}{36}.$$

The graph is



8. Note that, in factored form,

$$f(x) = \frac{(x+4)(x-1)}{(x+2)(x-1)}.$$

(a) The domain is the set of all real numbers except -2 and 1.

(b) $x = -2$

(c) Yes (the degree of numerator and denominator are the same): $y = 1$

(d) $y = 2$

(e) $x = -4$

(f)