Math 1110 Test 3. Fall 2016

1. List the rational numbers that are possible zeros of the function $f(x) = 2x^3 + x^2 - 6x - 12$, according to the rational zeros theorem. You do *not* need to verify whether the numbers in your list are actually zeros of the function.

2. According to Descartes' rule of signs, what are the possible numbers of positive real zeros that $f(x) = x^4 + 2x^3 - 3x^2 - x + 6$ can have (list all possibilities, not just the maximum)? What are the possible numbers of negative real zeros that $f(x)$ can have?

3. Find all real zeros of the function $f(x) = x^3 - 4x^2 + 9$.

4. Suppose that $f(x) = 3x^2 + 1$ and $g(x) = 2x - 5$.

(a) What is $f \circ g(0)$?

(b) What is $g \circ f(2)$?

5. Suppose that $f(x) = x^2 + 2x$ and that $g(x) = x^2 + 1$.

(a) Compute and simplify $f \circ g(x)$.

(b) Compute and simplify $g \circ f(x)$.

6. Suppose that $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{2}{x+3}$.

(a) Compute and simplify $f \circ g(x)$. Your final answer should not contain a compound fraction.

(b) What is the domain of $f \circ g(x)$?

7. (a) Find the inverse of $f(x) = 2x^3 + 1$.

(b) Find the inverse of $f(x) = \dfrac{2x+1}{x+3}$.

8. Suppose that $f(x)$ is a one-to-one function whose domain is $[2, 5]$ and whose range is $[3, 7]$.

(a) What is the domain of $f^{-1}(x)$?

(b) What is the range of $f^{-1}(x)$?

9. Use transformations to draw the graphs of the following functions. Clearly label asymptotes and intercepts.

(a) $F(x) = 2^x - 3$

(b) $G(x) = -(e^x) + 1$

(c) $f(x) = \ln(x + 2)$

(d) $g(x) = \log_2(-x)$

10. Solve each equation for x.

(a) $2^x = \dfrac{1}{16}$

(b) $3^{x^2} \cdot 9^{2x} = 3^5$

(c) $\log_8 64 = x + 1$

(d) $x \cdot \log_3 27 = \log_3 9$.

SOLUTIONS

1. The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 2 are 1 and 2. The possible rational zeros are

$$\pm\{1,\ 2,\ 3,\ 4,\ 6,\ 12,\ 1/2,\ 3/2\}.$$

2. $f(x)$ has two sign changes, so $f(x)$ has two or zero positive real zeros.

$$f(-x) = x^4 - 2x^3 - 3x^2 + x + 6$$

has two sign changes, so $f(x)$ has two or zero negative real zeros.

3. The possible rational zeros are $\pm\{1,\ 3,\ 9\}$. Test, using synthetic division. 1 doesn't work, but 3 does:

$$x^3 - 4x^2 + 9 = (x-3)(x^2 - x - 3).$$

The remaining quadratic doesn't factor, so use the quadratic formula

$$x = \frac{1 \pm \sqrt{1+12}}{2}.$$

The zeros of $f(x)$ are

$$\left\{3, \frac{1 \pm \sqrt{13}}{2}\right\}.$$

4. (a) $\quad f(g(0)) = f(-5) = 76$

(b) $\quad g(f(0)) = g(13) = 21$

5. (a)

$$\begin{aligned} f \circ g(x) &= f(x^2+1) \\ &= (x^2+1)^2 + 2(x^2+1) \\ &= x^4 + 2x^2 + 1 + 2x^2 + 2 \\ &= x^4 + 4x^2 + 3 \end{aligned}$$

(b)

$$\begin{aligned} g \circ f(x) &= g(x^2+2x) \\ &= (x^2+2x)^2 + 1 \\ &= x^4 + 4x^3 + 4x^2 + 1 \end{aligned}$$

6. (a)

$$\begin{aligned} f \circ g(x) &= f\left(\frac{2}{x+3}\right) \\ &= \frac{1}{\frac{2}{x+3} + 1} \\ &= \frac{1}{\frac{2}{x+3} + \frac{x+3}{x+3}} \\ &= \frac{1}{\frac{x+5}{x+3}} \\ &= \frac{x+3}{x+5} \end{aligned}$$

(b) x can not be -3 (because it is not in the domain of g). And x can not be a solution to

$$\frac{2}{x+3} = -1 \implies 2 = -x - 3 \implies x = -5$$

(for then $g(x)$ is not in the domain of f). The domain of the composition is the set of all real numbers except -3 and -5.

7. (a) Switch x and y. Solve for y.

$$\begin{aligned} x &= 2y^3 + 1 \\ x - 1 &= 2y^3 \\ \frac{x-1}{2} &= y^3 \\ y &= \sqrt[3]{\frac{x-1}{2}} \end{aligned}$$

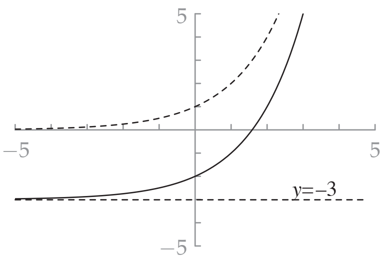
(b) Switch x and y. Solve for y.

$$\begin{aligned} x &= \frac{2y+1}{y+3} \\ x(y+3) &= 2y + 1 \\ xy + 3x &= 2y + 1 \\ xy - 2y &= 1 - 3x \\ y(x-2) &= 1 - 3x \\ y &= \frac{1-3x}{x-2} \end{aligned}$$

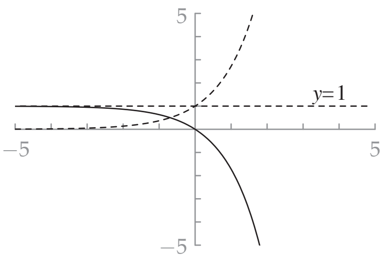
8. (a) $[3, 7]$. (b) $[2, 5]$

9.

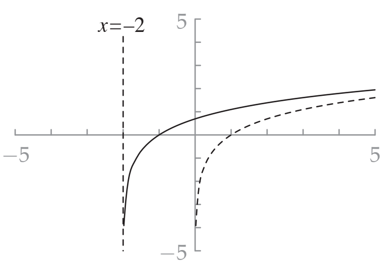
(a)



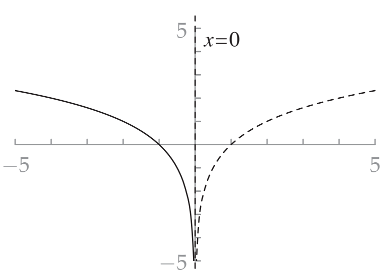
(b)



(c)



(d)



10. (a)
$$2^x = 2^{-4} \implies x = -4.$$

(b)

$$\begin{gathered}
3^{x^2} \cdot (3^2)^{2x} = 3^5 \\
3^{x^2} \cdot 3^{4x} = 3^5 \\
3^{x^2+4x} = 3^5 \\
x^2 + 4x = 5 \\
x^2 + 4x - 5 = 0 \\
(x-1)(x+5) = 0 \\
x + 1 = 0, \quad x + 5 = 0 \\
x = -1, \quad x = -5
\end{gathered}$$

(c)

$$\begin{gathered}
\log_8 64 = x + 1 \\
2 = x + 1 \\
1 = x
\end{gathered}$$

(d)

$$\begin{gathered}
x \cdot \log_3 27 = \log_3 9 \\
x \cdot 3 = 2 \\
x = 2/3
\end{gathered}$$