Use a Riemann sum to compute the area of the region above the x-axis, below the curve $y = x^3$, and between $x = 1$ and $x = 3$.

$$\Delta x = \frac{3-1}{n} = \frac{2}{n}$$

$$x_i = a + \Delta x i = 1 + \frac{2i}{n}$$

$$f(x_i) = \left(1 + \frac{2i}{n}\right)^3 = 1 + \frac{6i}{n} + \frac{12i^2}{n^2} + \frac{8i^3}{n^3}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{6i}{n} + \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right) \cdot \frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n} + \frac{12i}{n^2} + \frac{24i^2}{n^3} + \frac{16i^3}{n^4}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\sum_{i=1}^{n} 1 + \frac{12}{n^2}\sum_{i=1}^{n} i + \frac{24}{n^3}\sum_{i=1}^{n} i^2 + \frac{16}{n^4}\sum_{i=1}^{n} i^3\right)$$

$$\overset{*}{=} \lim_{n\to\infty} \left(\frac{2}{n} \cdot n + \frac{12}{n^2} \cdot \frac{n(n+1)}{2} + \frac{24}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{16}{n^4} \cdot \frac{n^2(n+1)^2}{4}\right)$$

$$= \lim_{n\to\infty} \left(2 + \frac{6(n+1)}{n} + \frac{4(n+1)(2n+1)}{n^2} + \frac{4(n+1)^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \left(2 + \frac{6n+6}{n} + \frac{8n^2+6n+4}{n^2} + \frac{4n^2+8n+4}{n^2}\right)$$

$$= 2 + 6 + 8 + 4$$

$$= 20$$

* summation formulas

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$