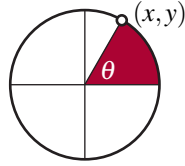


# Trigonometric Identities

## DEFINITIONS

$$\begin{aligned}\sin \theta &= y & \csc \theta &= 1/y \\ \cos \theta &= x & \sec \theta &= 1/x \\ \tan \theta &= y/x & \cot \theta &= x/y\end{aligned}$$



## PYTHAGOREAN IDENTITIES

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ \cot^2 \theta + 1 &= \csc^2 \theta\end{aligned}$$

## RECIPROCAL

$$\begin{aligned}\csc \theta &= \frac{1}{\sin \theta} & \tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta}\end{aligned}$$

## NEGATIVES

$$\begin{aligned}\sin(-\theta) &= -\sin(\theta) & \csc(-\theta) &= -\csc(\theta) \\ \cos(-\theta) &= \cos(\theta) & \sec(-\theta) &= \sec(\theta) \\ \tan(-\theta) &= -\tan(\theta) & \cot(-\theta) &= -\cot(\theta)\end{aligned}$$

## COFUNCTIONS

$$\begin{aligned}\sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta \\ \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta & \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta \\ \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta\end{aligned}$$

## ADDITION

$$\begin{aligned}\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\end{aligned}$$

## DOUBLE ANGLE

$$\begin{aligned}\sin(2\theta) &= 2 \sin \theta \cos \theta \\ \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ \tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

## HALF ANGLE

$$\begin{aligned}\sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{\theta}{2} &= \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}\end{aligned}$$

## POWER REDUCTION

$$\begin{aligned}\sin^2 \theta &= \frac{1 - \cos(2\theta)}{2} \\ \cos^2 \theta &= \frac{1 + \cos(2\theta)}{2} \\ \tan^2 \theta &= \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}\end{aligned}$$

## SUM TO PRODUCT

$$\begin{aligned}\sin \alpha + \sin \beta &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \sin \alpha - \sin \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha + \cos \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha - \cos \beta &= -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)\end{aligned}$$

## PRODUCT TO SUM

$$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]\end{aligned}$$